Worksheet 05 - Special Derivatives

Section 05

Instructions

This worksheet covers the differentiation of exponential, logarithmic, and trigonometric functions. For each problem, find the derivative and simplify your result. Difficulty levels are indicated by (x) = basic, (xx) = moderate, (xxx) = challenging, (xxxx) = advanced.

Problem Set 1: Exponential Functions

1. Basic Exponential Derivatives (x)

Find the derivative of each function:

a) $f(x) = e^{3x}$

b) $f(x) = 5e^{x}$

c) $f(x) = e^{-x}$

d) $f(x) = 3^{x}$

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2. Exponential Functions with Chain Rule (xx)

Find the derivative of each function:

a) $f(x) = e^{x^2}$

b) $f(x) = e^{2x+1}$

c) $f(x) = 2^{3x}$

d) $f(x) = e^{-x^2/2}$

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3. Products and Quotients with Exponentials (xxx)

Find the derivative of each function:

a) $f(x) = x^2 \cdot e^{x}$

b) $f(x) = \frac{e^{x}}{x}$

c) $f(x) = x \cdot e^{-x}$

d) $f(x) = \frac{e^{2x}}{e^{x}+1}$

Problem Set 2: Logarithmic Functions

4. Basic Logarithmic Derivatives (x)

Find the derivative of each function:

a) $f(x) = \ln(x)$

b) $f(x) = 3\ln(x)$

c) $f(x) = \log_{10}(x)$

d) $f(x) = \log_2(x)$

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5. Logarithmic Functions with Chain Rule (xx)

Find the derivative of each function:

a) $f(x) = \ln(3x)$

b) $f(x) = \ln(x^2 + 1)$

c) $f(x) = \ln(\sqrt{x})$

d) $f(x) = \log_3(2x + 5)$

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6. Products and Quotients with Logarithms (xxx)

Find the derivative of each function:

a) $f(x) = x \cdot \ln(x)$

b) $f(x) = \frac{\ln(x)}{x}$

c) $f(x) = \ln(x)^2$

d) $f(x) = x^2 \cdot \ln(x) - \frac{x^2}{2}$

Problem Set 3: Trigonometric Functions

7. Basic Trigonometric Derivatives (x)

Find the derivative of each function:

a) $f(x) = \sin(x)$

b) $f(x) = \cos(x)$

c) $f(x) = \tan(x)$

d) $f(x) = 3\sin(x) - 2\cos(x)$

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8. Trigonometric Functions with Chain Rule (xx)

Find the derivative of each function:

a) $f(x) = \sin(3x)$

b) $f(x) = \cos(x^2)$

c) $f(x) = \tan(2x)$

d) $f(x) = \sin(2x + \pi)$

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9. Products and Quotients with Trig Functions (xxx)

Find the derivative of each function:

a) $f(x) = x \cdot \sin(x)$

b) $f(x) = \frac{\sin(x)}{x}$

c) $f(x) = e^x \cdot \cos(x)$

d) $f(x) = \frac{\sin(x)}{\cos(x)}$ (verify this gives the derivative of $\tan(x)$)

Problem Set 4: Mixed Functions

10. Combining All Function Types (xxx)

Find the derivative of each function:

a) $f(x) = e^x \cdot \ln(x)$

b) $f(x) = \sin(x) \cdot \ln(x)$

c) $f(x) = e^{\sin(x)}$

d) $f(x) = \ln(\cos(x))$

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11. Business Applications (xxxx)

a) A company's revenue follows $R(t) = 500 \cdot e^{0.03t}$ thousand euros, where t is in years. Find the rate of change of revenue at $t = 5$ years and interpret it.

b) The demand for a product is modeled by $D(p) = 1000 \cdot e^{-0.02p}$, where p is the price in euros. Find $D'(p)$ and evaluate it at $p = 50$. What does the sign tell you?

c) A seasonal product has monthly sales modeled by $S(t) = 200 + 80\sin\left(\frac{\pi t}{6}\right)$, where t is the month (January = 1). Find $S'(t)$ and determine in which month sales are growing fastest.

Problem Set 5: Challenge Problems

12. Advanced Derivatives (xxxx)

Find the derivative of each function:

a) $f(x) = \ln\left(\frac{x^2+1}{x^2-1}\right)$

b) $f(x) = e^x \cdot \sin(x) \cdot \cos(x)$

c) $f(x) = x^x$ (Hint: rewrite using $e^{\ln(\cdot)}$)

d) $f(x) = \frac{\ln(x^2)}{e^{2x}}$