

FSP Training 05

Preparation for Mock Exam 06



Instructions

This training page covers all of Section 05 and prepares you for Mock Exam 06. Set A trains setting up: you translate a text into mathematics and check your translation. Set B is a practice mock in the format of Mock Exam 06: 180 minutes plus 5 minutes reading time, 100 points, and all 3 problems must be completed. Write down every step as you would in the exam. The points show what each part is worth.

In the real FSP, decimals are written with a comma (1,25 means 1.25).

Set A: Set Up & Check (40 minutes)

From Text to Mathematics (xx)

Translate each statement into mathematics. Then answer the check question: it tells you whether your translation works.

1. An FSP task says: “Investigate the asymptotic behavior of $f(x) = \frac{2x^2-18}{x^2-2x-3}$.” **List** every limit you have to compute for this task and **compute** them.

Check: **Compute** $f(2.99)$, $f(3.01)$, $f(-1.01)$ and $f(-0.99)$. At which of the two zeros of the denominator does the graph have a vertical asymptote, and what happens at the other one?

2. An FSP true/false task contains the statement: “The tangential gradient of f at $x = 1$ is bigger than the slope of the secant in the interval $[1; 3]$.” Here $f(x) = x^3 - 4x^2 + 6x$. The tangential gradient is the slope of the tangent. **Write** the statement as an inequality.

Check: **Compute** both sides of your inequality. Is the statement true or false?

3. An FSP task says: “Compute the first derivative of $f(x) = (4 - x) \cdot e^{0.5x}$ using the product rule.” **Write down** u , v , u' and v' , and **give** $f'(x)$ with $e^{0.5x}$ factored out.

Check: **Compute** $f'(0)$ with your result and with the d/dx template (**SHIFT** + **f**) at $x = 0$. Do both values agree?

4. An FSP task says: “Investigate the asymptotic behavior of $g(x) = (x + 5) \cdot e^{-0.4x}$.” A student writes: “ $e^{-0.4x}$ tends to 0, so $g(x) \rightarrow 0$ for $x \rightarrow \infty$ and for $x \rightarrow -\infty$.” **Write down** both limits correctly and **give** the horizontal asymptote.

Check: **Compute** $g(20)$ and $g(-10)$. Do the values fit your two limits?

5. An FSP task says: “Determine the nature (local maximum, local minimum or saddle point) and the coordinates of all stationary points of $f(x) = 0.25x^4 - 3x^3 + 12x^2 - 16x$.” A classmate has already factored the derivative: $f'(x) = (x - 1)(x - 4)^2$, then

writes: " $f''(4) = 0$, so $x = 4$ is not a stationary point." **Write down** the sign chart of f' and **decide** the nature of both stationary points.

Check: **Compute** $f'(3)$ and $f'(5)$. Does f' change its sign at $x = 4$?

6. "A company's monthly total costs are $C(x) = 0.5x^3 - 5x^2 + 20x + 120$ MU for x QU." An FSP task says: "Determine the short-term lower limit price." The short-term lower limit price is the lowest value of the variable costs per unit (the variable costs $C(x) - 120$ divided by the output x): below this price, the company cannot even cover its variable costs. **Write down** the function you have to minimize and **compute** the short-term lower limit price.

Check: **Compute** the variable costs per unit at 4 QU and at 6 QU. Are both higher than your price? A classmate minimizes $\frac{C(x)}{x}$ instead. Which FSP price does the classmate compute?

7. An FSP true/false task contains the statement: " f' possesses a maximum at $x = 3$." **Write down** the conditions on f'' (or f''') that make this statement true, and **name** the point of the graph of f at $x = 3$.

Hint: Use the second derivative test one level up: f' is the function, f'' its derivative and f''' its second derivative.

Check: **Test** the statement for $f(x) = -x^3 + 9x^2 - 21x + 2$. Is $x = 3$ a stationary point of f ?

8. An FSP task says: "The graph of a polynomial function f of third degree has the inflectional tangent $y = 3x - 1$ at $x = 1$." **Write down** every condition this sentence gives as an equation in a, b, c and d , with $f(x) = ax^3 + bx^2 + cx + d$.

Check: **Show** that both $f_1(x) = -0.5x^3 + 1.5x^2 + 1.5x - 0.5$ and $f_2(x) = x^3 - 3x^2 + 6x - 2$ fulfil your conditions. What does this tell you about the number of conditions you still need?

9. "The marginal costs of a company are $C'(x) = 0.6x^2 - 4x + 15$ MU per QU, and its fixed costs are 45 MU." An FSP task says: "Determine the cost function." **Give** $C(x)$.

Hint: Undo the power rule term by term: raise each exponent by 1 and divide by the new exponent.

Check: **Differentiate** your $C(x)$. Do you get $C'(x)$ back? How high are $C(0)$ and the costs of 5 QU?

One Paragraph, All Conditions (xx)

A company builds pressure washers (machines that clean terraces and cars with a strong jet of water) and sells them to DIY stores. Its monthly total costs are a polynomial of third degree in the output x . The fixed costs are 150,000 EUR per month. The short-term lower limit price (the lowest value of the variable costs per unit) is 200 EUR per pressure washer, and the variable costs per unit reach this lowest value at an output of 400 pressure washers. At an output of 600 pressure washers, the marginal costs are 340 EUR per pressure washer. The DIY stores buy 1,200 pressure washers per month at a price of 450 EUR each, and every price increase of 30 EUR costs 100 pressure washers in sales.

The demand is linear. The company can build at most 1,500 pressure washers per month. The owner wants to know the output with the highest profit, a proof that it really is a maximum, and the long-term price floor (the lowest value of the unit costs, the total costs per pressure washer).

(*QU = quantity units, MU = monetary units. 1 QU = 100 pressure washers, 1 MU = 1,000 EUR, so a price of p MU per QU is a price of $10p$ EUR per pressure washer*)

List every condition and every function in this paragraph as a mathematical statement, with x in QU and all money in MU. Do not solve anything.

Check: An assistant writes the slope of the price-demand function as $-\frac{1}{30}$. **Name** both mistakes.

FSP Words So Far

The FSP problems use these words from Sections 01 to 05. Learn what each one means and how to write it in mathematics.

Word	Meaning	Mathematics
quantity unit (QU)	a fixed number of items, e.g. 1 QU = 100 items	the output x is counted in QU
monetary unit (MU)	a fixed amount of money, e.g. 1 MU = 100 EUR	results in MU, convert to EUR at the end
fixed costs	costs that arise even if nothing is produced	the constant term of $C(x)$
variable costs	costs that grow with the output	$C(x) - C(0)$ (for linear costs: output times costs per QU)
total costs	fixed costs + variable costs	$C(x)$
revenue	quantity times price	$R(x) = x \cdot \text{price}$
profit	revenue – total costs	$P(x) = R(x) - C(x)$
break-even point	the output at which the profit changes from negative (a loss) to positive	$P(x) = 0$, the smaller zero
profit limit (upper break-even point)	the larger output with zero profit: above it, the profit turns negative again	$P(x) = 0$, the larger zero
profit zone	all outputs with a positive profit	$P(x) > 0$: from the break-even point to the profit limit (or the capacity)
capacity	the maximum possible output	an upper bound: $x \leq \text{capacity}$

Word	Meaning	Mathematics
growth factor	the factor by which a quantity is multiplied each period	q in (starting value) $\cdot q^t$
growth rate	the change per period in percent	$q = 1 + r$ with r as a decimal, a loss of 4 % gives $q = 0.96$
doubling time	the time after which a quantity has doubled	$q^T = 2$, continuous: $e^{kT} = 2$, so $T = \frac{\ln 2}{k}$
half-life (halving time)	the time after which a quantity has halved	$q^T = 0.5$, continuous: $e^{-kT} = 0.5$, so $T = \frac{\ln 2}{k}$
“deviates from a by at most d ” (tolerance)	the distance from a is at most d	$ x - a \leq d \Leftrightarrow a - d \leq x \leq a + d$
“share of A among B ”	the part of B that also belongs to A : the base is B	$\frac{\text{number in } A \cap B}{\text{number in } B}$
“neither A nor B ”	not in A and not in B , i.e. outside both circles	$\overline{A \cup B}$
contingency table	a table of counts for two properties: the inner fields are the regions of the Venn diagram, the margins are the totals	inner fields $A \cap B, A \cap \overline{B}, \overline{A} \cap B, \overline{A} \cap \overline{B}$
gross price, net price	gross price = net price + VAT	with 19 % VAT: gross = $1.19 \cdot$ net
system of linear equations	several linear equations that must hold at the same time	solve by substitution or elimination, check the solution in every equation
no solution, infinitely many solutions	the conditions contradict each other, or one condition only repeats another	elimination leaves $0 = d$ with $d \neq 0$, or $0 = 0$
demand, supply	the quantity buyers want to buy and the quantity sellers offer at a given price	as terms in the price p , e.g. $Q_d(p), Q_s(p)$, or as prices $p_D(x), p_S(x)$ for the quantity x
market equilibrium	the quantity demanded equals the quantity supplied	demand = supply: $Q_d(p) = Q_s(p)$, or $p_D(x) = p_S(x)$
price-demand function	gives the price at which x QU can be sold	$p(x)$, revenue $R(x) = x \cdot p(x)$
average cost	total costs divided by the number of units	$\frac{C(x)}{x}$ with $x \neq 0$

Word	Meaning	Mathematics
zero form	all terms on one side, 0 on the other side	$ax^2 + bx + c = 0$, then factor or use the quadratic formula
discriminant	decides how many real solutions a quadratic equation has	$\Delta = b^2 - 4ac$: > 0 two, $= 0$ one, < 0 none
biquadratic equation	an equation with only x^4 , x^2 and a constant	substitute $u = x^2$ in $ax^4 + bx^2 + c$, solve for u , then $x = \pm\sqrt{u}$ for $u \geq 0$
domain restrictions (of an equation)	the conditions the unknown must meet so that every term is defined	denominators $\neq 0$, under a root ≥ 0 , inside a logarithm > 0
extraneous solution	a candidate that solves the squared equation but not the original one	check every candidate in the original equation
function	assigns to every input exactly one output	$f(x)$, its graph passes the vertical line test
domain (of a function)	all inputs for which the function is defined, in a context only the sensible ones	e.g. $[0; \text{capacity}]$
range	all outputs that the function takes on its domain	if the function grows or falls: between the values at the two ends, for a parabola also check the vertex
maximum price	the price at which nobody buys any more	$Q_d(p) = 0$, or c in $p = c - dQ$
market saturation	the quantity demanded at a price of 0	a in $Q_d = a - bp$, or the zero of $p(x)$
shortage, surplus	at a price below the equilibrium price, demand exceeds supply (shortage), above it, supply exceeds demand (surplus)	$Q_d(p) > Q_s(p)$, or $Q_d(p) < Q_s(p)$
contribution margin	price minus variable costs per unit: the part of the price that covers the fixed costs	break-even quantity = $\frac{\text{fixed costs}}{\text{contribution margin}}$
vertex	the highest or lowest point of a parabola	$x_v = -\frac{b}{2a}$, or (h, k) in $a(x - h)^2 + k$
profit-maximizing output	the output with the highest profit	x_v of $P(x)$, a maximum since $a < 0$, and the price $p(x_v)$. If

Word	Meaning	Mathematics
		the capacity lies below x_v : the capacity and its price
shift, stretch, compression	moving or scaling a graph	$a \cdot f(b(x - h)) + k$: first b , then h , then a , then k
mirror image at the y -axis	the graph flipped from right to left	$f(-x)$
mirror image at the x -axis	the graph flipped upside down	$-f(x)$
composite function	first the inner function, then the outer function	$(f \circ g)(x) = f(g(x))$
inverse function	reverses a function: gives the input that belongs to an output	solve $y = f(x)$ for x , e.g. $x(p)$ from $p(x)$
polynomial of third degree	a function with powers of x up to x^3	$ax^3 + bx^2 + cx + d$ with $a \neq 0$: four unknowns, so four conditions
degree, leading coefficient	the highest power of x , and the number in front of it	together they decide the end behavior
zero of multiplicity m	the factor $(x - c)$ appears m times in the factored form	odd m : the graph crosses the x -axis at c , even m : it touches it
polynomial long division	divides a polynomial by $(x - c)$ once c is a known zero	$f(x) = (x - c) \cdot q(x)$, the remainder must be 0
asymptotic behavior (end behavior)	what happens to $f(x)$ for $x \rightarrow \infty$ and $x \rightarrow -\infty$	polynomial: degree and sign of the leading coefficient. Rational function: its asymptotes
intercepts	the points where the graph meets the axes	y -intercept $(0, f(0))$, x -intercepts from $f(x) = 0$
rational function	a quotient of two polynomials	$\frac{p(x)}{q(x)}$ with the domain $q(x) \neq 0$
vertical asymptote	a vertical line near which the values grow without bound	a zero of the denominator that does not cancel
hole	a single missing point of the graph	a factor that cancels, its y -value comes from the simplified term
horizontal asymptote	a horizontal line that the graph approaches for $x \rightarrow \pm\infty$	$y = c$ with $f(x) \rightarrow c$

Word	Meaning	Mathematics
oblique asymptote	a slanted line that the graph approaches for $x \rightarrow \pm\infty$	the linear part of the long division, if the numerator's degree is one higher
grows less than proportionally	grows, but twice the input gives less than twice the output	$a \cdot x^b$ with $0 < b < 1$
continuous growth rate	the rate k of a model with the base e	$N_0 \cdot e^{kt}$, growth factor per period e^k
saturation level	a level that a quantity approaches but never exceeds	a horizontal asymptote $y = L$, e.g. of $L - a \cdot e^{-kt}$ or $\frac{L}{1 + Ae^{-kt}}$
behavior for $t \rightarrow \infty$	the value a quantity approaches in the long run	$\lim_{t \rightarrow \infty} N(t)$, often with $e^{-kt} \rightarrow 0$
radian	an angle measure in which a full turn is 2π	$180^\circ = \pi$, arc length $s = r\theta$
amplitude	half the distance between the highest and the lowest value	$ A = \frac{\max - \min}{2}$
midline	the horizontal line halfway between the highest and the lowest value	$y = D = \frac{\max + \min}{2}$
period	the length of one full cycle	$B = \frac{2\pi}{\text{period}}$
phase shift	the horizontal shift of a sine or cosine curve	C in $A \sin(B(t - C)) + D$ or $A \cos(B(t - C)) + D$
mirror image at the midline	the graph flipped upside down about its midline $y = D$	$2D - f(t)$
limit	the value that $f(x)$ approaches when x approaches a number a , or $\pm\infty$	$\lim_{x \rightarrow a} f(x)$, one-sided: $x \rightarrow a^-$ (from the left), $x \rightarrow a^+$ (from the right)
continuous (at a)	the graph has no jump and no gap at a	$f(a)$ is defined, $\lim_{x \rightarrow a} f(x)$ exists, and both are equal
removable discontinuity	a gap that a single value could close, e.g. a hole	the limit exists, but $f(a)$ is not defined or differs from it
derivative	the instantaneous rate of change: the slope of the tangent	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
secant, slope of the secant	the line through two points of the graph, and its slope (the average rate of change)	$\frac{f(b) - f(a)}{b - a}$ on $[a; b]$

Word	Meaning	Mathematics
tangent, tangential gradient	the line that touches the graph at a point, and its slope	$y = f'(a)(x - a) + f(a)$ with the slope $f'(a)$
angle of intersection (with the x -axis)	the angle at which a line with slope m crosses the x -axis	$\alpha = \arctan(m)$, with the indicator D for degrees
marginal cost, marginal revenue, marginal profit	about the change of the costs, the revenue or the profit caused by one more unit	$C'(x), R'(x), P'(x) = R'(x) - C'(x)$
stationary point (critical point)	a point of the graph with a horizontal tangent	$f'(x_0) = 0$
nature of a stationary point	whether it is a local maximum, a local minimum or a saddle point	sign change of f' at x_0 , or the sign of $f''(x_0)$
local maximum, local minimum	a point that is higher (lower) than all points nearby	$f'(x_0) = 0$ and $f''(x_0) < 0$ (maximum) or $f''(x_0) > 0$ (minimum)
saddle point	a stationary point at which f' does not change its sign: the graph levels off and keeps its direction	$f'(x_0) = 0$ without a sign change, e.g. $f''(x_0) = 0$ and $f'''(x_0) \neq 0$
necessary condition, sufficient condition	a condition every candidate must meet, and a condition that proves the kind of point ("prove it really is a maximum")	necessary: $f'(x_0) = 0$, sufficient: e.g. $f''(x_0) < 0$ for a maximum
global maximum, global minimum (on an interval)	the highest or lowest value on the whole interval	compare the local extrema with the values at the ends of the interval
concave up, concave down	the graph curves upward like a cup, or downward like a cap	$f''(x) > 0$, or $f''(x) < 0$
inflection point	the point at which the curvature changes sign: the graph is steepest or flattest there	$f''(x_0) = 0$ and $f'''(x_0) \neq 0$, or a sign change of f''
"the curvature changes sign"	the graph changes from concave down to concave up, or back	an inflection point: $f''(x_0) = 0$. For a cubic cost function, the marginal costs are lowest there
inflectional tangent	the tangent at an inflection point: it shows the fastest growth or decline	$y = f'(a)(x - a) + f(a)$ with $f''(a) = 0$

Word	Meaning	Mathematics
unit costs, lowest unit costs	the total costs per unit, and the output at which they are lowest	$k(x) = \frac{C(x)}{x}$, minimum from $k'(x) = 0$ and $k''(x) > 0$
long-term price floor	the lowest unit costs: below this price, the company cannot cover its total costs in the long run	the minimum value of $k(x) = \frac{C(x)}{x}$
variable costs per unit	the variable costs divided by the output	$k_v(x) = \frac{C(x)-C(0)}{x}$
short-term lower limit price	the lowest variable costs per unit: below this price, the company cannot even cover its variable costs	the minimum value of $k_v(x)$
function family (German: <i>Funktionsschar</i>)	functions with the same term that differ only in a parameter	$f_a(x)$, e.g. $f_a(x) = x^2 - ax + 1$
parameter	a letter that is fixed within one function of the family but changes from one function to the next	solve a condition for it, e.g. $f'_a(2) = 0$

In set-builder notation, $\{x \in \mathbb{R} \mid \dots\}$ and $\{x \in \mathbb{R} : \dots\}$ mean the same.

Set B: Practice Mock (180 minutes, 100 points)

Problem 1: Digital Pianos for Music Retailers [35 pts. total]

A company builds digital pianos and sells them to music retailers. An analysis of its monthly costs gives the following information:

The fixed costs are 100 MU. At an output of 2 QU, the graph of the cost function changes from concave down to concave up. The tangent to the graph of the cost function at 6 QU is the line $y = 120x - 116$.

Nobody buys a digital piano at a price of 1,400 EUR or more (maximum price). At a price of 840 EUR per piano, the retailers buy 800 pianos per month. The demand is linear.

(1 QU = 100 digital pianos, 1 MU = 1,000 EUR, so a price of p MU per QU is a price of $10p$ EUR per piano. Round to two decimal places.)

- a) **Determine** the linear price-demand function $p(x)$, with p in MU per QU and x in QU. **Give** the market saturation in digital pianos per month. [3 pts.]

(for verification: $p(x) = 140 - 7x$)

- b) The total costs $C(x)$ in MU are a polynomial of third degree. **Translate** the three pieces of information about the costs into four equations, and **explain** why the sentence about the tangent gives two of them. **Solve** the system. [11 pts.]

(for verification: $C(x) = x^3 - 6x^2 + 84x + 100$)

- c) **Show** that the monthly profit is given by $P(x) = -x^3 - x^2 + 56x - 100$. [2 pts.]
- d) **Determine** the break-even point and the profit limit (upper break-even point). **Give** the number of digital pianos per month with which the company makes a profit.

Hint: The break-even point is a whole number of QU.

[6 pts.]

- e) **Determine** the output with the highest profit and **verify** with the second derivative that it is a maximum. **Give** the maximum profit in EUR and the price per digital piano that the company has to ask for. [5 pts.]
- f) The unit costs are the total costs per QU, $k(x) = \frac{C(x)}{x}$. The long-term price floor is the lowest value of the unit costs: below this price, the company cannot cover its total costs in the long run. **Compute** the output volume with the lowest unit costs. Also, **determine** the long-term price floor in EUR per digital piano.

Hint: Clear the fractions in $k'(x) = 0$ to get a cubic equation. It has exactly one real solution: test the divisors of 50.

[4 pts.]

- g) The variable costs per unit are the variable costs divided by the output, $k_v(x) = \frac{C(x)-100}{x}$. The short-term lower limit price is the lowest value of the variable costs per unit: below this price, the company cannot even cover its variable costs. **Compute** the short-term lower limit price in EUR per digital piano. A new competitor pushes the market price down to 800 EUR per piano. **Decide** whether the company should continue to produce in the short run, and **explain** whether it can cover its total costs in the long run at this price.

Hint: Compare the market price with both price limits. A price above the short-term lower limit price covers the variable costs and pays part of the fixed costs. If the company stops producing, it earns nothing but still has to pay the full fixed costs.

[4 pts.]

Problem 2: Analysis of an Exponential Function [40 pts. total]

Let the function f be given by

$$f(x) = (x^2 + 5x + 4) \cdot e^{-0.5x}, \quad x \in \mathbb{R}$$

Round all results to two decimal places.

- a) **State** the domain of f and **determine** the x - and y -intercepts of the graph of f . [3 pts.]

- b) **Determine** the limits of $f(x)$ for $x \rightarrow \infty$ and for $x \rightarrow -\infty$, and **give** the horizontal asymptote. **Describe** in one sentence how the graph of f behaves far to the right. [5 pts.]
- c) **Determine** the first and the second derivative of f . **Factor out** $e^{-0.5x}$ in both. [6 pts.]
(for verification: $f'(x) = -0.5(x+3)(x-2) \cdot e^{-0.5x}$ and $f''(x) = 0.25(x^2 - 3x - 8) \cdot e^{-0.5x}$)
- d) **Determine** the coordinates of all stationary points of f , and **decide** with the second derivative whether they are local maxima or local minima. [6 pts.]
- e) **Assess** how many inflection points f has to possess at least. **Give** a reason with your results from b) and d). A detailed computation is not necessary. [2 pts.]
- f) **Compute** the inflection points of f . [4 pts.]
- g) **Determine** the equation of the tangent t to the graph of f at its y -intercept. A line with slope m meets the x -axis at the angle $\alpha = \arctan(m)$. **Compute** the angle of intersection α between t and the x -axis.

Hint: The indicator **D** must show. **SHIFT + tan** gives $\arctan$, just as **SHIFT + sin** gives $\sin^{-1}$.

[5 pts.]

- h) **Argue** whether the following statements are true or false.
- (1) f has a global maximum at $x = 2$.
- (2) On the interval $[-1; 10]$, f takes its smallest value at $x = -1$.
- (3) The graph of f is concave down for every $x > 2$.
- (4) The graph of f is steeper at $x = -1$ than at $x = 0$.
- [4 pts.]
- i) **Sketch** the graph of f in the interval $[-4; 8]$ together with the tangent t from g). **Mark** the intercepts, the stationary points and the inflection points. [5 pts.]

Problem 3: A Sales Season and a Function Family [25 pts. total]

Part A: Easter Bunnies

A confectionery sells chocolate Easter bunnies in the 10 weeks before Easter. It models the weekly sales t weeks after the start of the season by a polynomial of third degree $S(t) = at^3 + bt^2 + ct + d$, with S in hundred bunnies per week. Last year's season gives the following information:

- At the start of the season ($t = 0$), the confectionery sells 2,000 bunnies per week.
- The weekly sales grow fastest two weeks after the start of the season.
- Six weeks after the start, the weekly sales reach their highest value of 12,800 bunnies per week.

- a) **Write down** the four equations for a , b , c and d that this information gives. **Explain** why the sentence about week 2 gives only one equation. [3 pts.]
- b) **Solve** the system of equations and **give** $S(t)$. [7 pts.]

(for verification: $S(t) = -0.5t^3 + 3t^2 + 18t + 20$)

- c) **Show** that $S(t) = -0.5(t - 10)(t + 2)^2$, and **explain** what the zero $t = 10$ means for the season. **Compute** by how much the weekly sales grow per week at the moment of the fastest growth. [3 pts.]

Part B: A Function Family

For every $k > 0$, the function g_k is given by

$$g_k(x) = e^x - kx, \quad x \in \mathbb{R}$$

- d) **Show** that every function g_k has exactly one stationary point, at $x = \ln(k)$. [2 pts.]
- e) **Prove** that this stationary point is a local minimum for every $k > 0$, and **show** that its y -value is $k(1 - \ln(k))$. **Show** that all graphs of the family pass through the same point. [3 pts.]
- f) **Determine** k so that the minimum lies at $x = 2$, and **give** the coordinates of this minimum. [2 pts.]
- g) **Determine** k so that the minimum lies on the x -axis, and **give** this lowest point. [2 pts.]
- h) **Explain** why g_k has no zero for $0 < k < e$, and why g_e has exactly one zero. [3 pts.]

Exam Preparation Checklist

Before the exam, ensure you can:

- ☐ Compute limits from a graph, by factoring and cancelling and for $x \rightarrow \pm\infty$, and check continuity with the three conditions
- ☐ Tell a hole (removable discontinuity) from a vertical asymptote with one-sided limits
- ☐ Compute the slope of a secant and the tangential gradient, and interpret marginal cost, marginal revenue and marginal profit
- ☐ Differentiate with the power, product, quotient and chain rules, including e^x , $e^{g(x)}$ and $\ln(x)$, and check a derivative value with the d/dx template
- ☐ Investigate the asymptotic behavior of functions like $(x^2 + bx + c) \cdot e^{-kx}$ with $x^n \cdot e^{-kx} \rightarrow 0$ for $x \rightarrow \infty$
- ☐ Determine the nature and the coordinates of all stationary points with the sign change of f' or the second derivative test, and recognize a saddle point
- ☐ Determine inflection points with f'' (sign change or $f''' = 0$), assess how many inflection points a function has to possess at least, and explain their meaning, e.g. the fastest growth or the lowest marginal costs
- ☐ Determine a tangent or an inflectional tangent and the angle of intersection $\alpha = \arctan(m)$ with the indicator **D**
- ☐ Find global extrema on a closed interval with the values at the ends, e.g. the highest profit within the capacity
- ☐ Sketch a graph with the 6-step algorithm and argue whether statements about f , f' and f'' are true or false
- ☐ Determine a cubic cost, revenue or sales function from conditions: an extremum or a tangent gives two equations, an inflectional tangent three

- ☐ Get the cost function from the marginal costs and the fixed costs
- ☐ Compute the maximum profit, prove that it is a maximum and give the price, and determine the break-even point and the profit limit
- ☐ Compute the lowest unit costs, the long-term price floor and the short-term lower limit price, and decide with them
- ☐ Work with a function family: stationary points in terms of the parameter, and the parameter for a given location or value

 Tip

Remember: Show all work, label units, and interpret your results in the business context.