Worksheet 05 - Complete Function Analysis

Section 05

Instructions for Complete Function Analysis

For each function, perform a complete function analysis including:

1. Domain: Identify all values of x for which the function is defined
2. Zeros: Find where $f(x) = 0$ (if they exist)
3. Y-intercept: Find $f(0)$ (if it exists)
4. Symmetry: Check for even symmetry ($f(-x) = f(x)$) or odd symmetry ($f(-x) = -f(x)$)
5. Asymptotes:
   - Vertical asymptotes (for rational functions)
   - Horizontal asymptotes (behavior as $x \to \pm\infty$)
   - Oblique/slant asymptotes (if applicable)
6. First derivative $f'(x)$:
   - Critical points (where $f'(x) = 0$ or undefined)
   - Intervals of increase/decrease
   - Local extrema (maxima and minima)
7. Second derivative $f''(x)$:
   - Inflection points (where $f''(x) = 0$ and changes sign)
   - Intervals of concavity (concave up where $f''(x) > 0$, concave down where $f''(x) < 0$)
8. Sketch: Draw the graph incorporating all the above information

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Problem Set 1: Polynomial Functions

Perform a complete function analysis for each of the following polynomial functions:

1. $f(x) = -\frac{1}{20}x^3 + 15x$

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2. $f(x) = \frac{1}{9}x^3 - \frac{1}{6}x^2 - 2x$

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3. $f(x) = 1.5x^4 + x^3 - 9x^2$

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4. $f(x) = x^3 - 6x^2 + 9x$

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5. $f(x) = -\frac{1}{20}x^4 + \frac{6}{5}x^2 - 4$

6. $f(x) = -\frac{1}{36}(3x^5 - 50x^3 + 135x)$

7. $f(x) = x^3 + 4x^2 - 11x - 30$

8. $f(x) = \frac{1}{9}x^5 - \frac{20}{27}x^4 + \frac{10}{9}x^3$

9. $f(x) = x^4 + x^3 - 11x^2 + 20$

10. $f(x) = \frac{1}{32}(5x^4 - x^5)$

Problem Set 2: Rational Functions

Perform a complete function analysis for each of the following rational functions:

1. $f(x) = \frac{8}{4-x^2}$

2. $f(x) = \frac{2-x^2}{x^2-9}$

3. $f(x) = \frac{x^2-4}{x^2+2}$

4. $f(x) = \frac{x^2}{(x-2)^2}$

5. $f(x) = \frac{x}{x^2+1}$

6. $f(x) = \frac{3x^2-3x}{(x-2)^2}$

7. $f(x) = \frac{3}{x} - \frac{12}{x^2}$

8. $f(x) = \frac{1}{2}x + \frac{1}{2} + \frac{2}{x^2}$

Problem Set 3: Function Families

For each function family (with parameter $t > 0$), perform a complete function analysis. Then sketch the graphs for $\boldsymbol{t = 1, 2, 3}$ in the same coordinate system.

1. $f_t(x) = x - \frac{t^3}{x^2}$ ____________________________

2. $f_t(x) = \frac{10}{x} - \frac{10t}{x^2}$ ____________________________

3. $f_t(x) = \frac{6x}{x^2+t^2}$ ____________________________

4. $f_t(x) = \frac{10x}{(x^2+t)^2}$ ____________________________

Additional Notes

i Working with Function Families

When analyzing function families: 1. First analyze the general case with parameter t 2. Identify how t affects key features (zeros, extrema, asymptotes) 3. Then substitute specific values ($t = 1, 2, 3$) to sketch 4. Use different colors or line styles to distinguish the three graphs

⚠ Common Mistakes to Avoid

- Forgetting to check the domain first (especially for rational functions)
- Missing vertical asymptotes where the denominator equals zero
- Confusing “no solution” with “zero” when finding zeros
- Not checking if critical points are actually extrema (use first or second derivative test)
- Forgetting to simplify before differentiating