Practice Worksheet

Transforming Functions

Transformation Patterns & Exam Techniques

Problem 1: Basic Transformation Identification (x)

For each function, identify all transformations from the base function:

a) $g(x) = 3\sin(x) - 2$ from $f(x) = \sin(x)$

b) $h(x) = 2^{x+1} + 3$ from $f(x) = 2^x$

c) $p(x) = \log_3(x - 4)$ from $f(x) = \log_3(x)$

d) $q(x) = -(x + 2)^2$ from $f(x) = x^2$

Problem 2: Quick Pattern Recognition (x)

Using the 4-step exam method, analyze the transformation: $g(x) = -3\cos(2x) + 1$ from $f(x) = \cos(x)$

Find: a) All transformations b) The new period c) The amplitude d) The range

Problem 3: Angle of Inclination (xx)

A line passes through points $A(0, 2)$ and $B\left(3, 2 + 3\sqrt{3}\right)$.

a) Find the slope m
b) Calculate the angle of inclination α with the positive x-axis
c) Write the equation of the line
d) Where does this line intersect $y = x^2$?

Problem 4: Multi-Step Transformations (xx)

Starting with $f(x) = \sqrt{x}$, apply these transformations in order: 1. Shift right 1 unit 2. Vertical stretch by factor 2 3. Reflect over x-axis 4. Shift up 3 units

a) Write the equation after each step
b) Write the final equation
c) Find the domain and range
d) Sketch both original and final functions

Problem 5: Exam-Style Application (xx)

A company's daily production follows:

$$P(t) = 100 + 20\sin\left(\frac{\pi}{6}(t - 3)\right)$$

where t is hours after 6 AM (so $t = 0$ at 6 AM).

a) What is the period of production?
b) When does maximum production occur?
c) What is the production range?
d) When is production exactly 100 units?

Problem 6: Logarithmic Transformations (xx)

Consider $g(x) = 2\log_3(3x) - 1$.

a) Simplify the expression using logarithm properties
b) Identify all transformations from $f(x) = \log_3(x)$
c) Find the domain
d) Find where $g(x) = 3$

Problem 7: Synthesis - Multiple Functions (xxx)

The height of a Ferris wheel car is modeled by:

$$h(t) = 25 - 20\cos\left(\frac{\pi}{15}t\right)$$

where h is height in meters and t is time in minutes.

Meanwhile, the number of riders waiting follows:

$$N(t) = 50 \cdot 2^{-t/30}$$

a) Find the period of the Ferris wheel
b) What are the minimum and maximum heights?
c) At what time is the car first at maximum height?
d) How many riders are waiting when the car first reaches maximum height?
e) When are there exactly 25 riders waiting?

Problem 8: Comprehensive Transformation Challenge (xxxx)

A company analyzes its profit using the transformed function:

$$P(x) = -400\left(\frac{1}{2}\right)^{x-3} + 800$$

where x is years since launch and P is profit in thousands.

a) Identify the base function and list all transformations applied
b) Find the horizontal asymptote and explain its business meaning
c) Calculate the profit at launch ($x = 0$)
d) When does the profit equal 600 thousand?
e) Sketch the function showing key features
f) The company wants to model seasonal variation by adding $100\sin(\pi x)$. What would be the new profit range after 3 years?