

# Worksheet 04 - Definition of Polynomial Functions

## Section 04: Polynomial Functions

### Definition of a Polynomial Function

A function  $f$  whose equation can be written in the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

is called a polynomial function of degree  $n$ . Here,  $a_0, a_1, \dots, a_n$  are real numbers,  $a_n \neq 0$ , and  $n$  is a natural number.

#### Note

The numbers  $a_0, a_1, \dots, a_n$  are called coefficients.

Counter-example: A function with the equation

$$f(x) = \frac{x + 0.5x^2}{x^2 - 2x + 1}$$

is not a polynomial function (it is a rational function).

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### Worked Examples

#### Example 1

A function with the equation

$$f(x) = 7x^4 - \sqrt{5}x + 1$$

is a polynomial of degree 4.

Coefficients:  $a_4 = 7, a_3 = 0, a_2 = 0, a_1 = -\sqrt{5}, a_0 = 1$ .

#### Example 2

A function with the equation

$$g(x) = x^2 - 4x - x^2$$

is a polynomial of degree 1 (simplifies to:  $g(x) = -4x$ ).

Coefficient:  $a_1 = -4, a_0 = 0$ .

#### Example 3

A function with the equation

$$h(x) = \frac{x^2}{x+1}$$

is not a polynomial function, because it cannot be written in the form

$$h(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$


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## Practice Problems

Instructions: For each function below, determine:

1. Is it a polynomial function? (Yes/No)
2. If yes, what is its degree?
3. If yes, list all non-zero coefficients

### Problem 1

$$f(x) = -4x^5 - 4$$


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### Problem 2

$$g(x) = x^{20} + 5x^5$$


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### Problem 3

$$h(x) = 2^x - 3x$$


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### Problem 4

$$i(x) = x^{-2} + 4x$$


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### Problem 5

$$j(x) = \frac{4}{x} + x$$


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### Problem 6

$$k(x) = 100$$


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### Problem 7

$$l(x) = (x-1)(x-3)$$


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### Problem 8

$$m(x) = \sqrt{2}x^2 - x + 1$$

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## Additional Practice Problems

For each function below, determine if it is a polynomial function and state its degree if applicable.

### Problem 9

$$n(x) = 3x^4 - 2x^3 + x^2 - 7$$

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### Problem 10

$$p(x) = \frac{x^3 + 2x}{x}$$

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### Problem 11

$$q(x) = x^{3/2} + 5x - 1$$

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## Challenge Problems

### Challenge 1

$$s(x) = (2x + 1)^3$$

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### Challenge 2

$$t(x) = \sqrt{x^4 + 2x^2 + 1}$$