

Recap Sheet

Logarithmic Functions & Applications

Logarithmic Functions - Problem Set

Problem 1: Basic Logarithm Evaluation (x)

Evaluate the following logarithms without a calculator:

- a) $\log_3(81)$
- b) $\log_5(125)$
- c) $\log_2\left(\frac{1}{16}\right)$
- d) $\log_{10}(0.01)$

Problem 2: Converting Between Forms (x)

Convert between exponential and logarithmic forms:

- a) Convert to logarithmic form: $4^3 = 64$
- b) Convert to logarithmic form: $10^{-1} = 0.1$
- c) Convert to exponential form: $\log_7(343) = 3$
- d) Convert to exponential form: $\log_2(32) = 5$

Problem 3: Using Logarithm Properties (xx)

Simplify the following expressions using logarithm properties:

- a) $\log_2(16) + \log_2(8)$
- b) $\log_5(625) - \log_5(25)$
- c) $3\log_4(2)$
- d) $\log_3(27x^3) - \log_3(9x)$

Problem 4: Solving Simple Exponential Equations (xx)

Solve the following exponential equations:

- a) $3^x = 27$
- b) $2^{x-1} = 16$
- c) $5 \cdot 2^x = 160$
- d) $4^{2x} = 64$

Problem 5: Solving Exponential Equations with Logarithms (xx)

Solve using logarithms (give exact answers and decimal approximations):

- a) $2^x = 10$
- b) $3^{x+1} = 20$
- c) $5^{2x} = 100$
- d) $1.08^t = 2$

Problem 6: pH Scale Application (xx)

The pH of a solution is given by $\text{pH} = -\log[H^+]$ where $[H^+]$ is the hydrogen ion concentration in moles/liter.

- a) Find the pH of a solution with $[H^+] = 10^{-5}$ moles/liter
- b) Find the pH of a solution with $[H^+] = 10^{-3.5}$ moles/liter
- c) If coffee has pH 5.0, what is its hydrogen ion concentration?
- d) How many times more acidic is a solution with pH 3 than one with pH 6?

Problem 7: Investment and Compound Interest (xxx)

Use logarithms to solve these compound interest problems:

- a) How long will it take €6,000 to grow to €10,000 at 5% annual interest compounded annually?
- b) An investment doubles in 10 years. What is the annual interest rate (compounded annually)?
- c) How long does it take to double your money at 7% annual interest compounded quarterly?
- d) Compare: Which is better - 6% compounded monthly for 10 years or 6.1% compounded annually for 10 years?

Problem 8: Richter Scale and Earthquake Intensity (xxxx)

The Richter scale magnitude is given by $M = \log(A/A_0)$ where A is the amplitude and A_0 is a reference amplitude.

- a) An earthquake has magnitude 6.5. Another earthquake has 50 times the amplitude. What is its magnitude?
- b) The 2011 Japan earthquake (magnitude 9.0) was how many times stronger in amplitude than a magnitude 6.0 earthquake?
- c) If earthquake A has magnitude 7.2 and earthquake B has magnitude 5.7, find the ratio of their amplitudes.

- d) Two earthquakes occur simultaneously with magnitudes 6.0 and 6.3. What would be the magnitude if their amplitudes were combined?