

FSP Training 02

Preparation for Mock Exam 03



Instructions

This training page covers all of Section 02 and prepares you for Mock Exam 03. Set A trains setting up: you translate a text into mathematics and check your translation. Set B is a practice mock in the format of Mock Exam 03: 90 minutes, 50 points, and all 3 problems must be completed. Write down every step as you would in the exam. The points show what each part is worth.

In the real FSP, decimals are written with a comma (1,25 means 1.25).

Set A: Set Up & Check (40 minutes)

From Text to Mathematics (xx)

Translate each statement into mathematics. Then answer the check question: it tells you whether your translation works. (*QU* = quantity units, *MU* = monetary units)

1. "A cinema sold 460 tickets on Saturday, regular tickets and reduced tickets. It sold 30 % more regular tickets than reduced tickets."

Check: Did the cinema sell 260 regular tickets?

2. "An office orders paper and ink cartridges twice. The first order, 3 boxes of paper and 2 cartridges, costs 260 EUR. The second order, 6 boxes of paper and 4 cartridges, costs 520 EUR. The office manager wants to find the price of one box and the price of one cartridge from these two orders."

Check: Do the two orders show that a box costs 60 EUR and a cartridge 40 EUR?

3. "In a market, the demand price is $p_D(x) = 30 - 2x$ (the price at which buyers take x thousand units) and the supply price is $p_S(x) = 6 + x$ (the price at which sellers offer x thousand units), both in EUR per unit. The market is in equilibrium (market equilibrium: buyers want to buy exactly the quantity that sellers offer, so at this quantity the demand price equals the supply price)."

Check: Is the equilibrium price 8 EUR per unit?

4. "The profit of a company is $P = -x^2 + 15x - 44$, with x in QU and P in 1,000 EUR. The owner claims that a profit of exactly 13,000 EUR is possible."

Check: Is the owner right?

5. "A company has the total costs $C = x^2 + 2x + 36$, with x in QU and C in MU. It sells 1 QU for 14 MU. At a break-even point, the revenue equals the total costs."

Check: Does the company make a profit at 5 QU?

6. “A company can sell x QU if it asks the price $p(x) = 30 - 3x$ MU per QU (price-demand function). Its revenue is 72 MU.”

Check: Is there only one output with this revenue?

7. “A car-sharing company has 40 cars, 12 of them electric. It buys x new electric cars and no other cars. Afterwards, at least 40 % of its cars are to be electric.”

Check: Are 4 new electric cars enough?

8. “A supplier of wooden pallets offers x QU at the supply price $p_S(x) = 4\sqrt{x+1}$ EUR per pallet. We look for the quantity at which the supply price is 12 EUR.”

Check: Does the supplier offer 35 QU at this price?

9. “An online shop has 300 customers in region A and 1,200 customers in region B. In region A, the number of customers grows by 32 % per year, in region B by 10 % per year. After t years, region A has more customers than region B.”

Check: Does region A have more customers than region B after 7 years?

One Paragraph, All Conditions (xx)

A family business builds wooden garden benches. It models its monthly total costs by a quadratic function $C(x) = ax^2 + bx + c$ of the output x in QU, with C in MU. Its fixed costs are 1,200 EUR per month. In a month with 20 benches, the total costs were 3,400 EUR, and in a month with 50 benches, they were 7,450 EUR. The business sells each bench for 190 EUR. It can build at most 60 benches per month. The owner also wants the average cost per bench (total costs divided by the number of benches) to be at most 165 EUR.

(1 QU = 10 benches, 1 MU = 100 EUR)

List every condition in this paragraph and the terms for the revenue and the profit, with x the monthly output in QU. **Give** the domain restriction of your average-cost condition. Do not solve anything.

Check: Which of your conditions rules out 20 benches per month?

FSP Words So Far

The FSP problems use these words from Sections 01 and 02. Learn what each one means and how to write it in mathematics.

Word	Meaning	Mathematics
quantity unit (QU)	a fixed number of items, e.g. 1 QU = 100 items	the output x is counted in QU
monetary unit (MU)	a fixed amount of money, e.g. 1 MU = 100 EUR	results in MU, convert to EUR at the end
fixed costs	costs that arise even if nothing is produced	the constant term of $C(x)$

Word	Meaning	Mathematics
variable costs	costs that grow with the output	output times costs per QU
total costs	fixed costs + variable costs	$C(x)$
revenue	quantity times price	$R(x) = x \cdot \text{price}$
profit	revenue – total costs	$P(x) = R(x) - C(x)$
break-even point	the output at which the profit changes from negative (a loss) to positive	$P(x) = 0$, the smaller zero
profit limit (upper break-even point)	the larger output with zero profit: above it, the profit turns negative again	$P(x) = 0$, the larger zero
profit zone	all outputs with a positive profit	$P(x) > 0$: from the break-even point to the profit limit (or the capacity)
capacity	the maximum possible output	an upper bound: $x \leq \text{capacity}$
growth factor	the factor by which a quantity is multiplied each period	q in (starting value) $\cdot q^t$
growth rate	the change per period in percent	$q = 1 + r$ with r as a decimal, a loss of 4 % gives $q = 0.96$
doubling time	the time after which a quantity has doubled	$q^T = 2$
half-life (halving time)	the time after which a quantity has halved	$q^T = 0.5$
“deviates from a by at most d ” (tolerance)	the distance from a is at most d	$ x - a \leq d \Leftrightarrow a - d \leq x \leq a + d$
“share of A among B ”	the part of B that also belongs to A : the base is B	$\frac{\text{number in } A \cap B}{\text{number in } B}$
“neither A nor B ”	not in A and not in B , i.e. outside both circles	$\overline{A \cup B}$
contingency table	a table of counts for two properties: the inner fields are the regions of the Venn diagram, the margins are the totals	inner fields $A \cap B, A \cap \overline{B}, \overline{A} \cap B, \overline{A} \cap \overline{B}$
gross price, net price	gross price = net price + VAT	with 19 % VAT: gross = $1.19 \cdot \text{net}$

Word	Meaning	Mathematics
system of linear equations	several linear equations that must hold at the same time	solve by substitution or elimination, check the solution in every equation
no solution, infinitely many solutions	the conditions contradict each other, or one condition only repeats another	elimination leaves $0 = d$ with $d \neq 0$, or $0 = 0$
demand, supply	the quantity buyers want to buy and the quantity sellers offer at a given price	as terms in the price P , or as prices $p_D(x)$, $p_S(x)$ for the quantity x
market equilibrium	the quantity demanded equals the quantity supplied	demand = supply, or $p_D(x) = p_S(x)$
price-demand function	gives the price at which x QU can be sold	$p(x)$, revenue $R(x) = x \cdot p(x)$
average cost	total costs divided by the number of units	$\frac{C(x)}{x}$ with $x \neq 0$
zero form	all terms on one side, 0 on the other side	$ax^2 + bx + c = 0$, then factor or use the quadratic formula
discriminant	decides how many real solutions a quadratic equation has	$\Delta = b^2 - 4ac$: > 0 two, $= 0$ one, < 0 none
biquadratic equation	an equation with only x^4 , x^2 and a constant	substitute $u = x^2$ in $ax^4 + bx^2 + c$, solve for u , then $x = \pm\sqrt{u}$ for $u \geq 0$
domain restrictions (of an equation)	the conditions the unknown must meet so that every term is defined	denominators $\neq 0$, under a root ≥ 0 , inside a logarithm > 0
extraneous solution	a candidate that solves the squared equation but not the original one	check every candidate in the original equation

In set-builder notation, $\{x \in \mathbb{R} \mid \dots\}$ and $\{x \in \mathbb{R} : \dots\}$ mean the same.

Set B: Practice Mock (90 minutes, 50 points)

Problem 1: Camping Tents [20 pts. total]

An outdoor equipment company produces camping tents. It models its monthly total costs by a quadratic function $C(x) = ax^2 + bx + c$. In March, it produced 200 tents at total costs of 15,800 EUR, and in April 600 tents at total costs of 23,000 EUR. In May, it produced 1,000 tents at an average cost (total costs divided by the number of tents) of

31.80 EUR per tent. Market research shows a linear price-demand function $p(x) = mx + n$, which gives the price at which x QU can be sold: at a price of 48 EUR per tent, the company sells 400 tents, and 1,200 tents bring a revenue of 43,200 EUR.

(QU = quantity units, MU = monetary units. 1 QU = 100 tents, 1 MU = 100 EUR)

- a) **Determine** the cost function $C(x)$ in MU, with x in QU, by setting up and solving a system of three linear equations.

(for verification: $C(x) = 0.5x^2 + 14x + 128$)

[6 pts.]

- b) **Determine** the price-demand function $p(x)$ in MU per QU by setting up and solving a system of two linear equations. **Give** the revenue function $R(x)$.

(for verification: $p(x) = 54 - 1.5x$)

[3 pts.]

- c) **Show** that the profit function is given by $P(x) = -2x^2 + 40x - 128$.

[2 pts.]

- d) **Show** by completing the square that $P(x) = -2(x - 10)^2 + 72$. **Use** this form to **determine** the (lower) break-even point and the profit limit (upper break-even point).

[4 pts.]

- e) **Use** the form from d) and the fact that a square is never negative to **determine** the maximum profit and the output at which it occurs. **Calculate** the price per tent the company must ask to sell exactly this output.

[3 pts.]

- f) A new warehouse lease raises the fixed costs from 128 MU to F MU, and all other terms stay the same. **Determine** with the help of the discriminant the largest value of F for which the company can still avoid a loss. **Interpret** the result for this value of F .

Hint: the company can still avoid a loss exactly if the equation $P(x) = 0$ still has a solution, i.e. if its discriminant is not negative.

[2 pts.]

Problem 2: Olive Oil Importer [17 pts. total]

An importer of olive oil supplies restaurants and fine-food shops.

- a) One delivery tour leads to a customer 180 km away: 150 km on the motorway, then 30 km on country roads. On country roads, the delivery van drives 40 km/h slower than on the motorway, and the whole trip takes 2 hours. Let v be the speed on the motorway in km/h (time = distance $\div$ speed). **Give** the domain restrictions and the condition that the situation adds. **Show** that the situation leads to the equation

$$v^2 - 130v + 3000 = 0$$

[4 pts.]

- b) **Solve** the equation. **Explain** why the equation has two solutions, but the situation only one. **Give** both speeds and the driving time on each part of the route.

[3 pts.]

- c) In the regional market for this olive oil, the demand price is $p_D(x) = 4\sqrt{10 - x}$ and the supply price is $p_S(x) = x + 2$ (x in 1,000 litres, p in EUR per litre). **Determine** the market equilibrium (quantity and price). **Explain** why one of the solutions of the squared equation does not describe an equilibrium.

[5 pts.]

- d) Four years ago, the importer bought a bottling machine for 25,600 EUR, and two years ago a labelling machine for 19,200 EUR. Both machines lose the same percentage of their value every year: each year, their values are multiplied by the same factor q . Today, the two machines are worth 18,900 EUR together. **Show** that q satisfies

$$256q^4 + 192q^2 - 189 = 0$$

and **determine** by how many percent the machines lose value every year.

[5 pts.]

Problem 3: Robot Lawn Mowers [13 pts. total]

A manufacturer of garden tools replaces its old robot lawn mower with a new model. t years after the launch of the new model, it sells $N(t)$ thousand new mowers and $O(t)$ thousand old mowers per year.

- a) At the launch, the new model sells 4,000 mowers per year, and two years later 9,000 mowers per year. **Determine** $N(t) = a \cdot b^t$. The old model sells 36,000 mowers per year at the launch, and its sales fall by one third every year. **Show** that $O(t) = 36 \cdot 1.5^{-t}$.

(for verification: $N(t) = 4 \cdot 1.5^t$)

[3 pts.]

- b) **Determine** from which time on the new model sells more than the old one, exactly as a logarithm and rounded to two decimal places.

[3 pts.]

- c) Both models are made on the same production line. **Determine** the times at which the total sales $S(t) = N(t) + O(t)$ are exactly 30,000 mowers per year, exactly and rounded to two decimal places.

Hint: substitute $u = 1.5^t$. Then $1.5^{-t} = \frac{1}{u}$.

[4 pts.]

- d) The production line only pays off as long as the total sales are at least 20,000 mowers per year. **Decide** with the discriminant whether the total sales ever fall to exactly 20,000 mowers per year. **Explain** why they always stay above this level.

Note: the total sales can only fall below 20,000 mowers per year at a time where they are exactly 20,000.

[3 pts.]

Exam Preparation Checklist

Before the exam, ensure you can:

- ☐ Solve systems of two linear equations by substitution or elimination, and systems of three by eliminating one variable first
- ☐ Decide whether a system has exactly one solution, no solution or infinitely many solutions
- ☐ Set up a cost function or a price-demand function from data points with a system of equations
- ☐ Determine a market equilibrium (quantity and price)
- ☐ Bring an equation to zero form and solve quadratic equations by factoring, with the quadratic formula or by completing the square
- ☐ Use the discriminant to decide how many solutions an equation has, also for an unknown parameter
- ☐ Solve biquadratic equations with the substitution $u = x^2$ and cubic equations by factoring
- ☐ Give the domain restrictions of a fractional equation and clear the denominators with the common denominator
- ☐ Solve radical equations and check every candidate in the original equation
- ☐ Solve exponential equations and inequalities with logarithms or with the substitution $u = a^x$
- ☐ Find break-even points and profit zones, convert QU and MU, and decide which solutions fit the situation
- ☐ Complete the square (with the target form given) and use that a square is never negative to find a maximum

Tip

Remember: Show all work clearly, label units, and always interpret your results in the business context!