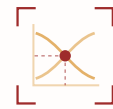


FSP Training 01

Preparation for Mock Exam 02



Instructions

This training page covers all of Section 01 and prepares you for Mock Exam 02. Set A trains setting up: you translate a text into mathematics and check your translation. Set B is a practice mock in the format of Mock Exam 02: 90 minutes, 50 points, and all 3 problems must be completed. Write down every step as you would in the exam. The points show what each part is worth.

In the real FSP, decimals are written with a comma (1,25 means 1.25).

Set A: Set Up & Check (40 minutes)

From Text to Mathematics (xx)

Translate each statement into mathematics. Then answer the check question: it tells you whether your translation works. (*QU* = quantity units, *MU* = monetary units)

1. "A car dealer sold 480 cars last year, 120 of them electric cars. Companies bought 200 of the 480 cars, among them 90 electric cars. We look at the share of electric cars among the cars bought by companies."

Check: Is this share the same as the share of company purchases among the electric cars?

2. "A coffee machine costs 214.20 EUR including 19 % VAT (value added tax), i.e. its gross price is the net price plus 19 % of the net price. In a sale, the shop takes 10 % off the net price and then adds the VAT."

Check: Does the customer pay the same as with 10 % off the gross price?

3. "The diameter d of a screw may deviate from 8 mm by at most 0.5 %."

Check: Is a screw with a diameter of 8.05 mm accepted?

4. "The cooling system of a server room raises an alarm as soon as the room temperature T deviates from 21 °C by more than 1.5 °C."

Check: Does the system raise an alarm at 19 °C?

5. "A square advertising banner has an area of 8 m². A smaller version of the banner, also square, has half of this area."

Check: Is the side length of the smaller banner half of the old side length?

6. "A company with the profit $(x - 2)(14 - x)$ (in 100 EUR) at an output of x QU and a capacity (the maximum possible output) of 12 QU makes a profit."

Check: Does $x = 13$ belong to this set?

7. "The number of customers who still use an old version of an app falls by 20 % per month. The half-life T is the time after which only half of these customers are left."
Check: After 3 months, are slightly more than half of these customers left?
8. "A language-learning app has 1,800 users today, and the number of users grows by 6 % per month. The team wants to know after how many months t the app reaches 9,000 users."
Check: Does the app reach 9,000 users within two years?
9. "A garage mixes 25 litres of windscreen washer fluid from water and a concentrate that contains 60 % alcohol. The fluid must contain at least 36 % alcohol."
Check: Are 12 litres of concentrate enough?
10. "A print shop makes T-shirts. Its fixed costs are 3.6 MU per month. Producing 1 QU costs another 0.9 MU, and the shop sells 1 QU for 1.5 MU." ($1 \text{ QU} = 100 \text{ T-shirts}$, $1 \text{ MU} = 1,000 \text{ EUR}$)
Check: What is the profit in EUR if the shop makes and sells 700 T-shirts?

One Paragraph, All Conditions (xx)

A kayak rental on a lake plans its July. Its fixed costs are 2,100 EUR for the month. They arise even if no kayak is rented. Each rental causes variable costs (they grow with the number of rentals) of 6 EUR for cleaning and wear. A customer pays 23.80 EUR per rental, including 19 % VAT. The rental keeps only the net price. With its boats, it can handle at most 600 rentals in July. The owner expects 450 rentals, but the actual number may deviate from this by at most 20 %.

List every condition in this paragraph and the terms for the total costs, the revenue and the profit, with x the number of rentals in July. Do not solve anything.

Check: Which of your conditions rules out 550 rentals, and which one does 550 rentals still meet?

FSP Words So Far

The FSP problems use these words from Section 01. Learn what each one means and how to write it in mathematics.

Word	Meaning	Mathematics
quantity unit (QU)	a fixed number of items, e.g. 1 QU = 100 items	the output x is counted in QU
monetary unit (MU)	a fixed amount of money, e.g. 1 MU = 100 EUR	results in MU, convert to EUR at the end
fixed costs	costs that arise even if nothing is produced	the constant term of $C(x)$
variable costs	costs that grow with the output	output times costs per QU

Word	Meaning	Mathematics
total costs	fixed costs + variable costs	$C(x)$
revenue	quantity times price	$R(x) = x \cdot \text{price}$
profit	revenue – total costs	$P(x) = R(x) - C(x)$
break-even point	the output at which the profit changes from negative (a loss) to positive	$P(x) = 0$, the smaller zero
profit limit (upper break-even point)	the larger output with zero profit: above it, the profit turns negative again	$P(x) = 0$, the larger zero
profit zone	all outputs with a positive profit	$P(x) > 0$: from the break-even point to the profit limit (or the capacity)
capacity	the maximum possible output	an upper bound: $x \leq \text{capacity}$
growth factor	the factor by which a quantity is multiplied each period	q in (starting value) $\cdot q^t$
growth rate	the change per period in percent	$q = 1 + r$ with r as a decimal, a loss of 4 % gives $q = 0.96$
doubling time	the time after which a quantity has doubled	$q^T = 2$
half-life (halving time)	the time after which a quantity has halved	$q^T = 0.5$
“deviates from a by at most d ” (tolerance)	the distance from a is at most d	$ x - a \leq d \Leftrightarrow a - d \leq x \leq a + d$
“share of A among B ”	the part of B that also belongs to A : the base is B	$\frac{\text{number in } A \cap B}{\text{number in } B}$
“neither A nor B ”	not in A and not in B , i.e. outside both circles	$\overline{A \cup B}$
contingency table	a table of counts for two properties: the inner fields are the regions of the Venn diagram, the margins are the totals	inner fields $A \cap B, A \cap \overline{B}, \overline{A} \cap B, \overline{A} \cap \overline{B}$
gross price, net price	gross price = net price + VAT	with 19 % VAT: gross = $1.19 \cdot \text{net}$

In set-builder notation, $\{x \in \mathbb{R} \mid \dots\}$ and $\{x \in \mathbb{R} : \dots\}$ mean the same.

Set B: Practice Mock (90 minutes, 50 points)

Problem 1: Designer Chairs [20 pts. total]

A furniture workshop produces designer chairs. Its fixed costs amount to 75 monetary units (MU). If it produces x quantity units (QU), the variable costs amount to $x^2 - 6x + 20$ MU per QU. To sell all x QU, the workshop can only charge a price of $45 - 3x$ MU per QU.

(1 QU = 100 chairs, 1 MU = 1,000 EUR)

- a) **Give** terms for the total costs $C(x)$ and the revenue $R(x)$.

(for verification: $C(x) = x^3 - 6x^2 + 20x + 75$, $R(x) = -3x^2 + 45x$)

[5 pts.]

- b) **Show** that the profit function is given by $P(x) = -x^3 + 3x^2 + 25x - 75$.

[3 pts.]

- c) **Factor** $P(x)$ completely by grouping. **Determine** all outputs at which the profit is exactly zero.

Note: a product is zero if one of its factors is zero. The smaller of these outputs is called the (lower) break-even point, the larger one the profit limit (upper break-even point).

(for verification: $P(x) = (x - 3)(5 - x)(5 + x)$)

[5 pts.]

- d) **Calculate** $P(2)$, $P(4)$ and $P(6)$. **Decide** in which output range the workshop makes a profit and **state** it as an interval and in set-builder notation.

(for verification: the workshop makes a profit exactly for outputs between 3 QU and 5 QU)

[3 pts.]

- e) **Calculate** the price per chair in EUR and the profit in EUR for an output of 450 chairs.

[2 pts.]

- f) The workshop plans an output of 4.5 QU, but the actual output deviates by up to 0.6 QU. **Formulate** this as an absolute value inequality, **solve** it and **decide** whether a profit is guaranteed. **Determine** up to which deviation a profit is still guaranteed.

[2 pts.]

Problem 2: Company Vans [16 pts. total]

A delivery company buys a van for 48,000 EUR ($t = 0$, t in years). Two years later, the van is worth 34,680 EUR. The van loses the same percentage of its value every year.

- a) **Determine** the yearly growth factor q and the percentage by which the value decreases per year. **Give** the function $V(t)$ for the value of the van.

(for verification: $V(t) = 48000 \cdot 0.85^t$)

[3 pts.]

- b) **Compute** the value after ten years. **Give** the result in scientific notation with three significant digits (i.e. round the leading number to two decimal places, e.g. $1.23 \cdot 10^5$).

[2 pts.]

- c) **Determine** the time after which the van has lost half of its value, exactly as a logarithm and rounded to two decimal places.

[3 pts.]

- d) A second van costs 60,000 EUR at $t = 0$ and loses 20 % of its value every year. **Give** the function $W(t)$ for its value. **Show** that both vans have the same value at

$$t = \frac{\ln 5 - \ln 4}{\ln 17 - \ln 16}$$

and **determine** in which year this happens.

[5 pts.]

- e) The accountant prefers the model $V(t) = 48000 \cdot e^{-kt}$. **Determine** k exactly so that both models describe the same loss of value. **Explain** why k is not equal to 0.15.

[3 pts.]

Problem 3: Commuter Survey and Cleaning Agent [14 pts. total]

A logistics company with 1,200 employees asks how they get to work. 45 % of all employees use a company bike (B), 660 employees use a public transport ticket (T), and 300 employees use neither.

- a) **Draw** a Venn diagram and **complete** the following table.

	ticket (T)	no ticket ($\bar{T}$)	sum
bike (B)			
no bike ($\bar{B}$)			
sum			1200

(for verification: 300 employees use both offers)

[5 pts.]

- b) **Argue** whether the following statements are true or false.

1. "Every employee with a ticket also uses a company bike." ($x \in T \Rightarrow x \in B$)
2. $\bar{B} \cap \bar{T} = \overline{B \cup T}$
3. More than half of all employees use exactly one of the two offers.

[3 pts.]

- c) **Compute** the percentage of ticket holders who also use a company bike and the percentage of company bike users who do not have a ticket.

[2 pts.]

- d) For 60 litres of cleaning agent, the company mixes a 15 % concentrate (2 EUR per litre) with a 35 % concentrate (5 EUR per litre). The mixture must contain at least 20

% active ingredient, and the budget is at most 210 EUR. **Determine** all possible amounts of the 15 % concentrate.

[4 pts.]

Exam Preparation Checklist

Before the exam, ensure you can:

- ☐ Work with intervals, set operations and Venn diagrams
- ☐ Decide whether statements and implications are true
- ☐ Compute percentages with the correct base and chain growth factors
- ☐ Apply exponent laws and scientific notation
- ☐ Factor with common factors, grouping, the AC method, substitution and special products
- ☐ Simplify radicals and rationalize denominators
- ☐ Apply logarithm laws, the change of base formula and $\ln$
- ☐ Solve exponential equations with logarithms and convert to continuous growth e^{kt}
- ☐ Solve absolute value equations and inequalities
- ☐ Translate word problems into linear equations and inequalities (IDEA)
- ☐ Set up cost, revenue and profit functions and find break-even points

Tip

Remember: Show all work clearly, label units, and always interpret your results in the business context!