# Powers with Integer Exponents

## Doubling with a Look into the Past

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### Example: Doubling with a Look into the Past

Starting from our example:

| Time (in min.) $= t$ | Number of Cents $= C$ | Powers (new notation) |
|---|---|---|
| 0 | 1 | $2^0$ |
| 1 | 2 | $2^1$ |
| 2 | $2 \cdot 2 = 4$ | $2^2$ |
| 3 | $2 \cdot 2 \cdot 2 = 8$ | $2^3$ |
| ... | ... | ... |

Let's take a look into the past. If I have one cent at starting time "0", how many cents did I have one minute ago, two minutes ago, ...?

| Time (in min.) $= t$ | Number of Cents $= C$ | Powers (new notation) |
|---|---|---|
| $-4$ (4 minutes ago) | $\frac{1}{2^4} = \frac{1}{16}$ | $2^{-4}$ |
| $-3$ (3 minutes ago) | $\frac{1}{2^3} = \frac{1}{8}$ | $2^{-3}$ |
| $-2$ (2 minutes ago) | $\frac{1}{2^2} = \frac{1}{4}$ | $2^{-2}$ |
| $-1$ (1 minute ago) | $\frac{1}{2^1} = \frac{1}{2}$ | $2^{-1}$ |
| 0 | 1 | $2^0$ |
| 1 | 2 | $2^1$ |
| 2 | $2 \cdot 2 = 4$ | $2^2$ |
| 3 | $2 \cdot 2 \cdot 2 = 8$ | $2^3$ |
| ... | ... | ... |

> **Note**
>
> To describe growth uniformly, one defines:
>
> $2^{-1} = \frac{1}{2^1}; \quad 2^{-2} = \frac{1}{2^2}; \quad 2^{-3} = \frac{1}{2^3}, \ldots$

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## Definition: Powers with Negative Integer Exponents

For all rational (also real) numbers $a \neq 0$ and natural numbers $n \in \mathbb{N}$:

$$a^{-n} = \frac{1}{a^n}$$

> ⚠ Warning
>
> Note: $a \neq 0$, because division by zero is not allowed!

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## Exercises (without calculator)

1) Write as a power of ten (with negative exponents)
   a) $0.1 = \frac{1}{10} = \frac{1}{10^1} = 10^{-1}$
   b) $0.01 =$
   c) $0.001 =$
2) Write as a decimal number without powers
   a) $7 \cdot 10^{-3} = 7 \cdot 0.001 = 0.007$
   b) $1.2 \cdot 10^{-4} =$
   c) $10^{-7} =$
3) Write as a fraction without powers
   a) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$
   b) $\left(\frac{2}{5}\right)^{-3} =$
   c) $\left(\sqrt{2}\right)^{-2} =$
   d) $(-4)^{-3} =$