Exponential Functions

The Legend of Chess and Rice Grains

The Legend of Chess and Rice Grains

The Story

Long ago in ancient India, the game of chess was invented - a strategic game played on a board with 64 squares. The Indian Emperor was so delighted with this ingenious game that he wanted to reward its inventor generously.

The emperor told: “Ask for any reward you desire, but do not be too modest!”

The humble request surprised everyone: “My lord, I ask only for rice grains. Place 1 grain of rice on the first square of the chessboard, 2 grains on the second square, 4 grains on the third square, and for each following square, double the number of grains from the previous square.”

The emperor felt almost insulted by such a modest request - he had no idea what he was agreeing to!

Let’s Investigate This “Simple” Request

Pattern Recognition

Let’s see how the rice grains accumulate on the first few squares:

Square Number (n)	Rice Grains (a_n)	Calculation	Power Notation
1	1	1	2^0
2	2	1×2	2^1
3	4	2×2	2^2
4	8	4×2	2^3
5	16	8×2	2^4
6	32	16×2	2^5
7	?	?	?
8	?	?	?

Tip

Your Task: Complete the table for squares 7 and 8. Can you spot the pattern?

Finding the General Formula

From the pattern above, we can see that for square n, the number of rice grains is:

$$a_n = 2^{n-1}$$

This is called an exponential function with:

- First term: $a_1 = 1$
- Common ratio: $q = 2$ (we double each time)

The Reality

Calculate the 64th Square

Using our formula $a_n = 2^{n-1}$, let's find how many rice grains would be needed for the 64th (final) square:

$$a_{64} = 2^{64-1} = 2^{63}$$

Using a calculator: $2^{63} = 9,223,372,036,854,775,808$

Total Rice Grains

The total number of rice grains for all 64 squares would be:

$$S_{64} = a_1 + a_2 + a_3 + ... + a_{64} = 2^{64} - 1$$

$$S_{64} = 18,446,744,073,709,551,615$$

Putting This in Perspective

Weight Calculation

Let's estimate the actual weight:

- Approximately 100 rice grains weigh 3 grams
- So 1 rice grain weighs about 0.03 grams

For the 64th square alone:

$$\text{Weight} = 9,223,372,036,854,775,808 \times 0.03 \text{ g} \approx 277 \text{ billion tons}$$

For all 64 squares:

$$\text{Total Weight} \approx 553 \text{ billion tons}$$

Reality Check: In 2006, the entire world produced only 634 million tons of rice. The "modest" request would require about 873 years of the world's entire rice production!

Understanding Exponential Growth

Key Insight: This story demonstrates the incredible power of exponential growth. Even though each step seems small (just doubling), the cumulative effect becomes large very quickly.

! Important

This is why exponential functions are so important in mathematics - they appear in:

- Population growth
- Compound interest
- Radioactive decay
- Viral spread
- Computer processing power