

## Session 09-06: Tasks

### Final Recap - Function Analysis & Probability

#### Exam-Style Problem 1: Function Analysis

Consider the function

$$f(x) = -2x(x - 3)^2 + 4 \quad \text{and} \quad g(x) = -2x + 4$$

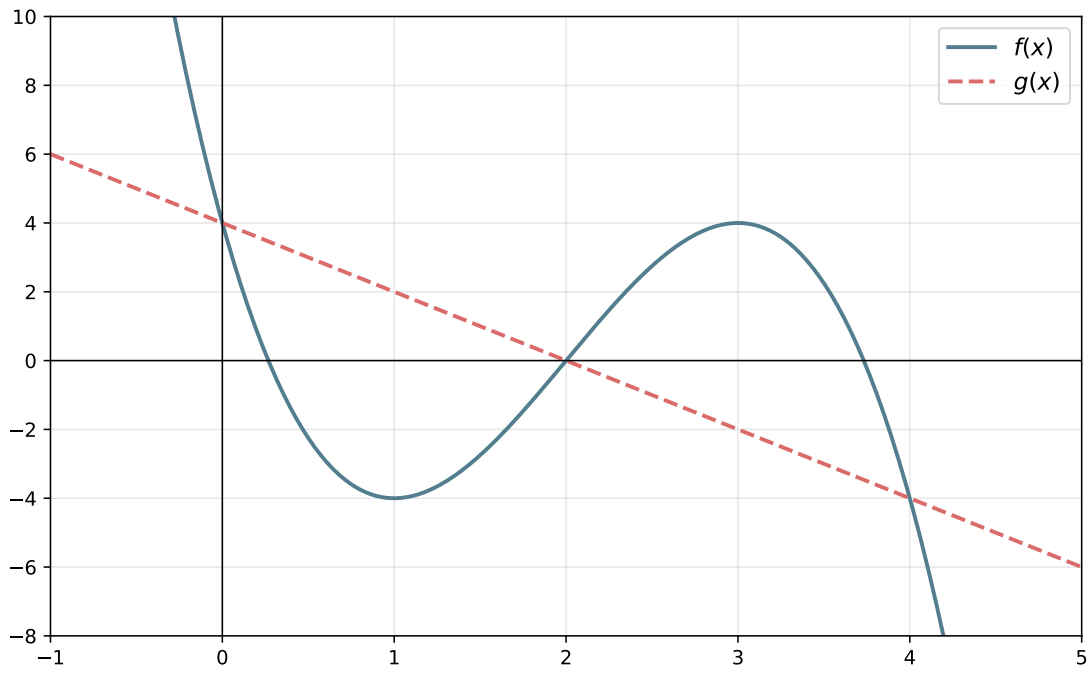


Figure 1: Graphs of  $f(x)$  and  $g(x)$

#### Part a)

Verify computationally that  $f(x)$  possesses a maximum at  $x = 3$ .

#### Part b)

Give a function  $f_1(x)$  which is a mirror image of  $f(x)$  at the  $y$ -axis and give a reason for your choice.

#### Part c)

The functions  $f(x)$  and  $g(x)$  enclose two regions. Show computationally that both regions have equal size.

#### Exam-Style Problem 2: Exponential Function

Consider the function  $j(x) = (x - 2) \cdot e^x + 1$ .

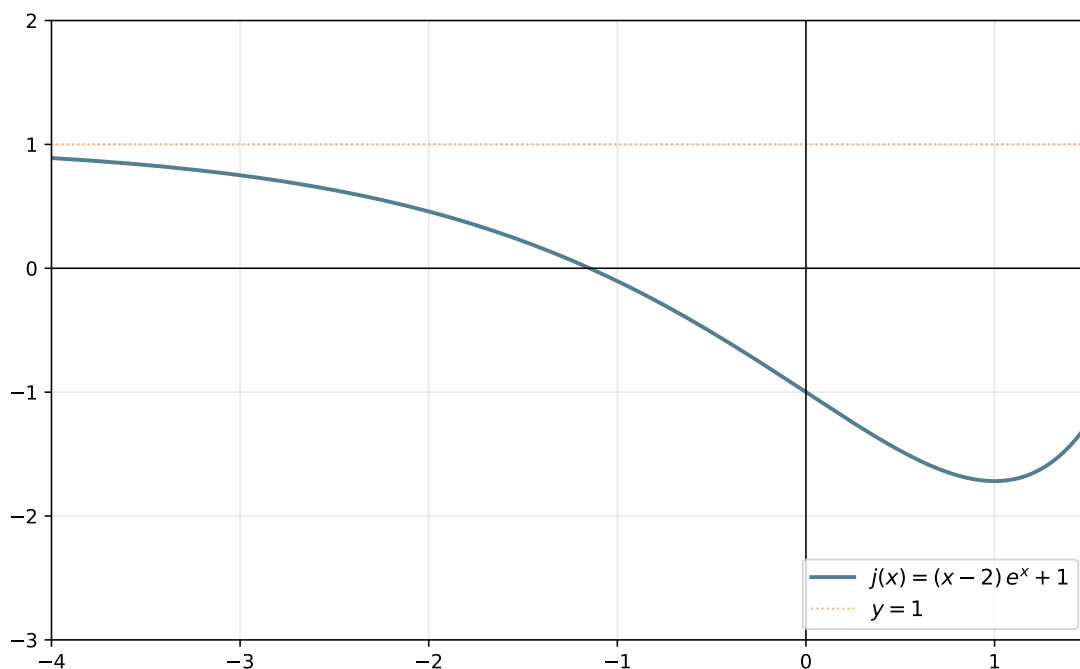


Figure 2: Graph of  $j(x) = (x - 2)e^x + 1$

Part a)

Determine the end behavior of  $j(x)$  at positive and negative infinity analytically.

Part b)

Compute the inflection point of  $j(x)$ .

Part c)

Compute the antiderivative of  $j(x) = (x - 2) \cdot e^x + 1$ .

### Exam-Style Problem 3: Probability & Statistics

A bookstore has a reading club for its customers.

Consider the following probabilities:

- The probability of a random person being a customer of this bookstore is 25%.
- A customer of this bookstore has an 80% chance of reading daily.
- A non-customer has a 15% chance of reading daily.

Notation:

- $C$ : “is a customer of this bookstore”
- $R$ : “reads daily”

Part a)

Display the fractions in a contingency table for a population of 100,000 people. Fill in the blank fields.

	$C$	$\bar{C}$	Sum
$R$			
$\bar{R}$			
Sum			100,000

**Part b)**

Compute the probabilities  $P(\bar{C} \cap R)$  and  $P(R | \bar{C})$ .

**Part c)**

A person reads daily. Compute the probability that this person is a customer of the bookstore.

**Part d)**

For a sample of 10 people determine the likelihood of the following events. This is a binomial distribution with  $n = 10$  and  $p = P(C) = 0.25$ .

$$P(X = k) = \binom{10}{k} (0.25)^k (0.75)^{10-k}$$

1. Exactly 3 persons are customers of this bookstore.
2. Not more than 3 persons are customers of this bookstore.
3. At least 2 but not more than 4 persons are customers of this bookstore.