Session 09-04: Tasks

Probability & Statistics - Exam Review

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Problem 1: Basic Probability (x)

a) A die is rolled twice. Find the probability that the sum is 7.

b) A bag contains 5 red, 3 blue, and 2 green balls. Find the probability of drawing a red or green ball.

c) Two cards are drawn from a standard 52-card deck without replacement. Find the probability both are aces.

d) Events A and B satisfy $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cap B) = 0.2$. Find $P(A \cup B)$ and $P(A \mid B)$.

Problem 2: Combinatorics (x)

a) In how many ways can 5 books be arranged on a shelf?

b) A team of 4 is chosen from 12 people. How many teams are possible?

c) A PIN consists of 4 digits (0–9). How many PINs are possible if digits can repeat? If digits cannot repeat?

Problem 3: Contingency Table Reading (x)

A survey of 200 students:

	Math Major	Science Major	Business Major	Total
Satisfied	30	25	45	100
Neutral	10	15	25	50
Dissatisfied	10	10	30	50
Total	50	50	100	200

a) Find $P(\text{Satisfied})$

b) Find $P(\text{Business Major} \mid \text{Dissatisfied})$

c) Find $P(\text{Dissatisfied} \mid \text{Math Major})$

d) Are "Satisfied" and "Math Major" independent? Check using the definition.

Problem 4: Contingency Table Construction (xx)

In a factory, 60% of products are from Machine A and 40% from Machine B. Machine A has a 4% defect rate, Machine B has a 6% defect rate.

a) Construct a contingency table for 500 products.

b) What is the overall defect rate?

c) A defective product is found. What is the probability it came from Machine A?

d) A non-defective product is found. What is the probability it came from Machine B?

Problem 5: Bayes' Theorem with Tree Diagram (xx)

A store has three suppliers:

- Supplier X: 50% of stock, 5% defective
- Supplier Y: 30% of stock, 3% defective
- Supplier Z: 20% of stock, 8% defective

a) Draw a tree diagram with all probabilities.

b) Find the overall defect rate.

c) An item is defective. Find the probability it's from Supplier Z.

d) An item is not defective. Find the probability it's from Supplier X.

Problem 6: Binomial Distribution Calculations (xx)

A multiple-choice exam has 15 questions, each with 4 options (one correct).

a) If a student guesses randomly, what distribution models the number correct? State parameters.

b) Find $P(X = 5)$.

c) Find $P(X \geq 3)$.

d) Find the expected number of correct answers and the standard deviation.

e) The pass mark is 8 correct. What is $P(\text{pass by guessing})$?

Problem 7: Descriptive Statistics from Frequency Table (xx)

Weekly overtime hours for 50 employees:

Hours	0–2	2–4	4–6	6–8	8–10
Frequency	8	14	16	9	3

a) Calculate the estimated mean.

b) Find the median class and estimate the median.

c) Calculate the standard deviation.

d) The company policy allows a maximum average of 5 hours overtime. Is the company in compliance?

Problem 8: Probability System of Equations (xx)

Events A and B satisfy:

- $P(A \cup B) = 0.8$
- $P(A \cap B) = 0.15$
- $P(A) = 2 \cdot P(B)$

a) Set up a system of equations.

b) Solve for $P(A)$ and $P(B)$.

c) Find $P(A \mid B)$ and $P(B \mid A)$.

d) Are A and B independent?

Problem 9: Conditional Probability Word Problem (xx)

At a university, 70% of students who study regularly pass the exam, while only 20% of those who don't study regularly pass. If 60% of students study regularly:

a) What is the overall pass rate?

b) Given that a student passed, what is the probability they studied regularly?

c) Given that a student failed, what is the probability they did not study regularly?

Problem 10: Expected Value Calculations (xx)

A game costs €5 to play. You roll two dice:

- Sum of 12: win €50
- Sum of 7: win €10
- Any other sum: win nothing

a) Find the probability distribution of winnings.

b) Calculate the expected winnings.

c) Calculate the expected profit (winnings minus cost).

d) If you play 100 times, what is your expected total profit/loss?

Problem 11: Quality Control Multi-Step Bayes (xxx)

A screening process has two stages:

- Stage 1 test: Sensitivity 90%, Specificity 85%
- Stage 2 test (only applied to Stage 1 positives): Sensitivity 95%, Specificity 92%
- Base rate of defects: 3%

a) What fraction of items test positive in Stage 1?

b) Among Stage 1 positives, what is the new "prevalence" of actual defects? (i.e., PPV of Stage 1)

c) What fraction of Stage 1 positives also test positive in Stage 2?

d) What is the final PPV after both stages?

e) How does the two-stage process compare to a single test with the same sensitivity and specificity as Stage 2?

Problem 12: Binomial + Expected Profit (xxx)

A company produces batches of 100 items. Each item has a 2% probability of being defective (independent).

a) What is the expected number of defective items per batch?

b) Each defective item costs €150 to replace. What is the expected replacement cost per batch?

c) The company offers a warranty: if more than 5 items in a batch are defective, the entire batch is replaced at a cost of €8,000. Find $P(X > 5)$.

d) What is the expected cost per batch including the warranty?

e) Should the company price the warranty at €200 per batch? Justify.

Problem 13: Insurance Pricing Word Problem (xxx)

An insurance company models car accident claims:

- 80% of drivers are "safe" with 2% annual accident probability
- 20% of drivers are "risky" with 10% annual accident probability
- Average claim: €8,000

a) What is the overall probability of a claim?

b) A driver files a claim. What is the probability they are "risky"? (Bayes)

c) The company charges a flat premium. What minimum annual premium ensures expected profit?

d) If the company charges €500/year, what is the expected profit per policyholder?

e) After a claim-free year, what is the updated probability that a policyholder is "risky"? (Bayes with updating)

Problem 14: "How Many Trials Needed" with Logarithms (xxx)

a) A biased coin has $P(\text{heads}) = 0.6$. How many flips are needed so that $P(\text{at least one head}) > 0.999$? Solve using logarithms.

b) A machine produces items with 1% defect rate. How many items must be inspected to be 95% sure of finding at least one defect?

c) A radioactive substance has a half-life of 5 years. After how many years is less than 1% of the original amount remaining? (Use $0.5^{t/5} < 0.01$ and logarithms.)

Problem 15: Sequential Bayes (xxx)

A coin is either fair ($P(H) = 0.5$) or biased ($P(H) = 0.8$). Initially, you believe there's a 50% chance the coin is biased.

a) You flip the coin and get heads. Update the probability the coin is biased.

b) You flip again and get heads. Update again.

c) You flip a third time and get tails. Update again.

d) After these 3 flips (H, H, T), what is the probability the coin is biased?

e) How many heads in a row would you need to be 99% sure the coin is biased?

Problem 16: Full Exam-Style Problem (xxxx)

A company produces electronic components on two assembly lines:

- Line A: 60% of production, 4% defect rate
- Line B: 40% of production, 7% defect rate

The company tests all components. The test has:

- Sensitivity: 92% (correctly detects defective)
- Specificity: 88% (correctly passes non-defective)

a) Construct a full contingency table for 10,000 components (include production line AND test result).

b) A component tests positive. What is the probability it is truly defective?

c) A component from Line A tests positive. What is the probability it is truly defective?

d) The company discards all test-positive items. What fraction of discarded items are actually good? (False positive rate among positives.)

e) Each false positive costs €20 (wasted good component). Each false negative costs €200 (defective reaches customer). What is the expected total quality cost per 10,000 components?

f) Would it be cost-effective to add a second test (at €0.50 per component) to retest all positives? The second test has the same sensitivity and specificity. Calculate and compare.

Problem 17: Multi-Stage Quality Assurance with Cost Optimization (xxxx)

A pharmaceutical company tests drug batches. Each batch either passes (good) or fails (contaminated). The prior probability of contamination is 5%.

Test characteristics: Sensitivity 98%, Specificity 90%.

a) Calculate the PPV after one test.

b) If a batch tests positive, it can be retested. Using the PPV from (a) as the new prior, calculate the PPV after a second positive test.

c) Each test costs €1,000. A contaminated batch that reaches market costs €500,000 in liability. A good batch destroyed costs €50,000 in production loss. Set up the expected cost for the following strategies:

- Strategy 1: No testing (ship all batches)
- Strategy 2: One test (discard if positive)
- Strategy 3: Two tests (discard only if both positive)

d) Which strategy minimizes expected cost?

e) At what contamination rate does Strategy 3 become cheaper than Strategy 2?