Session 09-03: Tasks

Integral Calculus - Exam Review

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Problem 1: Antiderivatives (x)

Find the antiderivative:

a) $\int (5x^4 - 3x^2 + 2)\,dx$

b) $\int e^{4x}\,dx$

c) $\int \frac{7}{x}\,dx$

d) $\int 6\sqrt{x}\,dx$

e) $\int \frac{3}{x^4}\,dx$

f) $\int \left(2e^{-x} + \frac{1}{x}\right)dx$

Problem 2: Definite Integrals (x)

Evaluate:

a) $\int_1^3 (2x+1)\,dx$

b) $\int_0^2 (x^2 - 2x)\,dx$

c) $\int_1^e \frac{2}{x}\,dx$

d) $\int_0^{\ln 2} e^{3x}\,dx$

Problem 3: Basic Area Under a Curve (x)

a) Find the area under $f(x) = x^2 + 1$ from $x = 0$ to $x = 3$.

b) Find the area under $f(x) = e^x$ from $x = 0$ to $x = 2$.

Problem 4: Area Between Two Curves (xx)

Find the area enclosed between:

a) $f(x) = 6 - x^2$ and $g(x) = x$ (find intersections first!)

b) $f(x) = x^2$ and $g(x) = \sqrt{x}$ on $[0, 1]$

Problem 5: Integration by Parts (xx)

Evaluate using integration by parts ($\int u\,dv = uv - \int v\,du$):

a) $\int x \cdot e^{3x}\,dx$

b) $\int x \cdot e^{-x}\,dx$

c) $\int \ln(x)\,dx$ (hint: let $u = \ln(x)$, $dv = dx$)

Problem 6: Consumer and Producer Surplus (xx)

A market has demand $D(q) = 80 - 4q$ and supply $S(q) = 20 + 2q$.

a) Find the equilibrium quantity and price.

b) Calculate consumer surplus.

c) Calculate producer surplus.

d) What is the total economic surplus?

Problem 7: Substitution Integrals (xx)

Evaluate using substitution:

a) $\int 2x \cdot e^{x^2}\,dx$

b) $\int \frac{3x}{x^2+4}\,dx$

c) $\int x(x^2+1)^4\,dx$

Problem 8: Continuous Income Present Value (xx)

An investment generates a continuous income stream of $f(t) = 2000 \cdot e^{0.03t}$ euros per year.

a) Find the total income over 5 years.

b) If the discount rate is $r = 0.06$, find the present value: $PV = \int_0^5 2000 \cdot e^{0.03t} \cdot e^{-0.06t}\,dt$

c) Is the investment worth more than 8,000 euros today?

Problem 9: Cost Function from Marginal Cost (xx)

A company's marginal cost is $MC(x) = 0.03x^2 - 2x + 50$ and fixed costs are 1,200 euros.

a) Find the total cost function $C(x)$.

b) Find the cost of producing the first 20 units.

c) Find the average cost function $\overline{C}(x) = \frac{C(x)}{x}$.

d) Find the production level that minimizes average cost (uses derivatives – Section 05 connection).

Problem 10: Average Value of a Function (xx)

The average value of a function f on $[a, b]$ is $\bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$.

a) Find the average value of $f(x) = x^2$ on $[0, 3]$.

b) Find the average value of $g(x) = e^{-x}$ on $[0, 4]$.

c) A company's production rate over 8 hours is $R(t) = 100 - 5t$ units/hour. Find the average production rate over the 8-hour period.

Problem 11: Area Between Exponential and Linear (xxx)

Find the area between $f(x) = e^x$ and $g(x) = x + 1$ on the interval $[0, 1]$.

a) Show that the curves intersect at $x = 0$.

b) Determine which function is on top on $(0, 1]$.

c) Set up and evaluate the integral for the enclosed area.

Problem 12: Waste Accumulation (xxx)

A chemical plant releases waste at a rate $R(t) = 80 \cdot e^{-0.2t}$ tons/year.

a) Find the total waste released in the first T years: $Q(T) = \int_0^T 80 \cdot e^{-0.2t}\, dt$.

b) Compute $Q(5)$ and $Q(20)$.

c) What value does $Q(T)$ approach as T becomes very large?

d) After how many years has 90% of the long-run total been released? (Use logarithms.)

e) Environmental regulations cap total waste at 350 tons. Will the plant comply in the long run?

Problem 13: Combined Curve Sketching + Area (xxx)

For $f(x) = x \cdot e^{-x}$:

a) Find zeros.

b) Find $\lim_{x\to\infty} f(x)$.

c) Find $f'(x)$, critical points, and classify them.

d) Find $f''(x)$ and inflection points.

e) Sketch the graph.

f) Find the area under $f(x)$ from $x = 0$ to $x = 3$: $\int_0^3 x \cdot e^{-x}\, dx$ (use integration by parts).

Problem 14: Revenue Optimization via Integration (xxx)

A firm's marginal revenue is $MR(x) = 200 - 4x$ and marginal cost is $MC(x) = 40 + 2x$.

a) Find the revenue function $R(x)$ if $R(0) = 0$.

b) Find the cost function $C(x)$ if fixed costs are 500 euros.

c) Find the profit function $P(x)$.

d) Find the profit-maximizing quantity (using derivatives).

e) Calculate the total profit at the optimal quantity.

f) Calculate the consumer surplus at the optimal quantity, given $D(x) = 200 - 4x$ is the demand curve.

Problem 15: Investment Comparison (xxx)

Compare two investment strategies over 10 years at discount rate $r = 0.05$:

- Strategy A: Lump sum of 50,000 euros invested today.
- Strategy B: Continuous deposit of 6,000 euros per year.

a) What is the present value of Strategy A? (Trivially 50,000 euros.)

b) What is the present value of Strategy B? Compute $PV_B = \int_0^{10} 6,000 \cdot e^{-0.05t}\, dt$.

c) Which strategy is more attractive?

d) At what annual deposit rate d (euros per year) would Strategy B have the same present value as Strategy A? Set up and solve the equation.

Problem 16: Surplus with Price Floor (xxxx)

A market has demand $D(q) = 120 - q^2$ and supply $S(q) = q^2 + 20$.

a) Find the free-market equilibrium (q^*, p^*).

b) Calculate consumer surplus and producer surplus at equilibrium.

c) The government imposes a price floor of $p_f = 80$. Find the new quantities demanded and supplied.

d) Calculate the deadweight loss caused by the price floor.

e) Interpret the redistribution of surplus.

Problem 17: Full Business Scenario (xxxx)

A monopolist faces demand $P(q) = 100 \cdot e^{-0.1q}$ and has cost function $C(q) = 200 + 10q$.

a) Find the revenue function $R(q) = q \cdot P(q)$.

b) Find the marginal revenue $MR(q)$ (product rule!).

c) Set $MR = MC$ and solve for the profit-maximizing quantity.

d) Find the monopoly price.

e) Calculate consumer surplus under monopoly pricing: $CS = \int_0^{q^*} P(q)\, dq - p^* \cdot q^*$.

f) If the competitive price were $P = MC = 10$, find the competitive quantity and compare total surplus.