

## Session 09-01: Tasks

### Advanced Functions - Exam Review

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##### Problem 1: Logarithm Rules (x)

Simplify the following expressions using logarithm properties:

- a)  $\log_3(27) + \log_3(9)$
- b)  $\ln(e^5 \cdot e^{-2})$
- c)  $\log_2(32) - \log_2(4)$
- d)  $\log_{10}(500) + \log_{10}(2)$

##### Problem 2: Exponential Equations (x)

Solve for  $x$ :

- a)  $5^{x+1} = 125$
- b)  $2^{3x} = 16$
- c)  $e^{2x-1} = e^5$

##### Problem 3: Asymptote Identification (x)

Find all vertical and horizontal asymptotes:

- a)  $f(x) = \frac{4}{x+3}$
- b)  $g(x) = \frac{3x-1}{x+2}$
- c)  $h(x) = \frac{x^2+1}{x^2-9}$
- d)  $k(x) = \frac{2x^3}{x^2-1}$

##### Problem 4: Domain and Range (x)

Find the domain and range:

- a)  $f(x) = \ln(x - 3)$
- b)  $g(x) = e^x + 2$
- c)  $h(x) = \frac{1}{x^2-4}$

##### Problem 5: Polynomial Factorization and Zeros (xx)

Factor completely and find all real zeros:

- a)  $p(x) = x^3 - 4x^2 + x + 6$
- b)  $q(x) = x^4 - 13x^2 + 36$
- c)  $r(x) = 2x^3 + 3x^2 - 8x + 3$

### Problem 6: Complete Rational Function Analysis (xx)

For  $f(x) = \frac{x^2 - 4x + 3}{x^2 - 5x + 6}$ :

- Factor numerator and denominator
- Identify any holes and vertical asymptotes
- Find the horizontal asymptote
- Find all x-intercepts and the y-intercept
- Sketch the graph

### Problem 7: Transformations and Inverse (xx)

Given  $f(x) = 2 \cdot \ln(x - 1) + 3$ :

- Describe all transformations from the base function  $\ln(x)$
- Find the domain and range
- Find the inverse function  $f^{-1}(x)$
- Verify that  $f(f^{-1}(x)) = x$

### Problem 8: Exponential Growth Word Problem (xx)

A bacteria population grows according to  $N(t) = 500 \cdot e^{0.15t}$ , where  $t$  is in hours.

- What is the initial population?
- How long until the population reaches 2000?
- What is the doubling time?
- At what rate is the population growing after 3 hours?

### Problem 9: Logarithmic Equations with Domain (xx)

Solve and check domain restrictions:

- $\log_3(x + 2) + \log_3(x) = 1$
- $\ln(2x + 1) - \ln(x - 2) = \ln(3)$
- $2 \log_5(x) = \log_5(9) + \log_5(x - 2)$

### Problem 10: Market Saturation Model (xxx)

A new product's market share is modeled by  $M(t) = \frac{60}{1 + 14 \cdot e^{-0.5t}}$  (logistic model, % share,  $t$  in months).

- What is the initial market share at  $t = 0$ ?
- What is the long-term equilibrium market share?
- When does the market share reach 30%?
- Show that  $M'(t) = \frac{420 \cdot e^{-0.5t}}{(1 + 14e^{-0.5t})^2}$  and find when the growth rate is maximized.
- Interpret the inflection point in business terms.

### Problem 11: Rational Function with Hole and Asymptotes (xxx)

For  $f(x) = \frac{x^3 - x^2 - 6x}{x^2 - 4}$ :

- Factor completely

- b) Identify holes, vertical asymptotes, and zeros
- c) Determine if there is a horizontal or oblique asymptote using polynomial long division
- d) Find the y-intercept (if it exists)
- e) Sketch the complete graph including the oblique asymptote

### Problem 12: Function Composition and Domain (xxx)

Let  $f(x) = \sqrt{x+4}$  and  $g(x) = \frac{1}{x-1}$ .

- a) Find  $(f \circ g)(x)$  and determine its domain
- b) Find  $(g \circ f)(x)$  and determine its domain
- c) Find  $x$  such that  $(f \circ g)(x) = 2$
- d) Is  $(f \circ g)(x) = (g \circ f)(x)$  for all  $x$  in the intersection of their domains? Prove or disprove.

### Problem 13: Oblique Asymptote Analysis (xxx)

For  $f(x) = \frac{2x^2+3x-5}{x+1}$ :

- a) Perform polynomial long division to find the oblique asymptote
- b) Find all zeros of  $f$
- c) Determine the vertical asymptote and behavior near it
- d) Find where  $f(x)$  intersects its oblique asymptote
- e) Sketch the graph showing the oblique asymptote clearly

### Problem 14: Competing Exponential Models (xxxx)

Two competing technologies have adoption rates:

- Technology A:  $A(t) = 100(1 - e^{-0.4t})$  (%)
- Technology B:  $B(t) = 100(1 - e^{-0.15t})$  (%)

where  $t$  is in years.

- a) Which technology is adopted faster initially? Justify using derivatives at  $t = 0$ .
- b) Find the exact time when both technologies have equal adoption rates.
- c) A company invests if a technology reaches 80% adoption. How much sooner does Technology A reach this threshold compared to Technology B?
- d) A “combined market” function is  $C(t) = A(t) + B(t) - A(t) \cdot B(t)/100$ . Find  $\lim_{t \rightarrow \infty} C(t)$  and interpret.

### Problem 15: Multi-Step Logarithmic Modeling (xxxx)

A company’s revenue follows  $R(t) = a \cdot \ln(bt + 1) + c$ , where  $t$  is years since founding. Given:  $R(0) = 2$  million,  $R(3) = 8$  million,  $R(10) = 12$  million.

- a) Set up a system of three equations to find  $a$ ,  $b$ , and  $c$ .
- b) Solve the system (show that  $c = 2$  immediately, then solve for  $a$  and  $b$ ).
- c) Predict  $R(20)$ .
- d) When will revenue reach 15 million?

- e) Find  $\lim_{t \rightarrow \infty} R(t)$  and discuss whether this model is realistic for long-term prediction.

### Problem 16: Comprehensive Function Analysis (xxxx)

For  $f(x) = \frac{(x-1)(x+2)}{(x-3)} \cdot e^{-x/2}$ :

- Find the domain
- Find all zeros
- Determine the behavior as  $x \rightarrow 3$  (from both sides)
- Find  $\lim_{x \rightarrow \infty} f(x)$  and  $\lim_{x \rightarrow -\infty} f(x)$
- Find  $f'(x)$  and determine all critical points (set up the equation — exact solution not required, but describe the method)
- Discuss what makes this function's graph particularly interesting compared to a standard rational function