Tasks 08-03 - Cost Analysis & Pricing Decisions

Section 08: Financial Mathematics

Problem 1: Identifying Cost Components (x)

For each cost function, identify the fixed cost and variable cost function:

a) $K(x) = 500 + 8x$
b) $K(x) = 1200 + 15x + 0.02x^2$
c) $K(x) = 2000 + 25x - 0.3x^2 + 0.01x^3$
d) $K(x) = 800 + 12x$

Problem 2: Variable Cost Per Unit (x)

Find the variable cost per unit function $k_v(x)$ for each cost function:

a) $K(x) = 600 + 10x$
b) $K(x) = 1000 + 20x + 0.05x^2$
c) $K(x) = 1500 + 30x - 0.2x^2 + 0.002x^3$
d) $K(x) = 400 + 5x + 0.1x^2$

Problem 3: Average Total Cost (x)

Find the average total cost function $\bar{K}(x)$ for each:

a) $K(x) = 800 + 6x$
b) $K(x) = 1200 + 18x + 0.03x^2$
c) $K(x) = 500 + 10x + 0.02x^2$

Problem 4: Short-Term Lower Limit Price (xx)

Find the short-term lower limit price (kurzfristige Preisuntergrenze) for each cost function:

a) $K(x) = 400 + 12x$ (linear variable costs)

b) $K(x) = 900 + 24x + 0.06x^2$

c) $K(x) = 1600 + 40x - 0.4x^2 + 0.004x^3$

Problem 5: Long-Term Lower Limit Price (xx)

Find the long-term lower limit price (langfristige Preisuntergrenze) for each cost function:

a) $K(x) = 800 + 10x + 0.02x^2$

b) $K(x) = 1200 + 15x + 0.05x^2$

c) $K(x) = 500 + 8x + 0.04x^2$

Problem 6: Both Lower Limits (xxx)

For the cost function $K(x) = 1000 + 20x - 0.1x^2 + 0.001x^3$:

a) Find the variable cost per unit function $k_v(x)$.
b) Find the short-term lower limit price.
c) Find the average total cost function $\bar{K}(x)$.
d) Find the long-term lower limit price.
e) Interpret the difference between the two limits.

Problem 7: Production Decisions (xx)

A company has cost function $K(x) = 600 + 15x + 0.03x^2$.

a) Find the short-term lower limit price.
b) Find the long-term lower limit price.
c) Should they accept an order for 100 units at 22 Euro each?
d) At what price would they break even when producing 100 units?

Problem 8: Special Order Analysis (xxx)

A manufacturer has cost function $K(x) = 2400 + 30x - 0.15x^2 + 0.001x^3$ and normally produces 80 units at a price of 45 Euro.

A new customer offers to buy 40 additional units (total would be 120) at 32 Euro each.

a) Find the short-term lower limit price.
b) Calculate the contribution margin per unit for the special order.
c) Calculate the total additional profit (or loss) from accepting the order.
d) Should the company accept? Why or why not?

Problem 9: Contribution Margin Analysis (xx)

A company has cost function $K(x) = 1500 + 12x + 0.04x^2$ and sells at p = 25 Euro.

a) Find the contribution margin per unit at production levels of 50, 100, and 150 units.
b) At what production level is the contribution margin zero?
c) What is the break-even quantity?

Problem 10: Complete Cost Analysis (xxx)

A factory produces widgets with cost function $K(x) = 3000 + 50x - 0.3x^2 + 0.002x^3$.

a) Find $K_v(x)$, $k_v(x)$, and $\bar{K}(x)$.
b) Find the short-term lower limit price and the quantity where it occurs.
c) Find the long-term lower limit price and the quantity where it occurs.
d) At what price should the company sell to maximize profit if they can produce up to 150 units?
e) Create a pricing decision chart showing the three zones.

Problem 11: Exam-Style Problem (xxxx)

Given: A company produces electronic components with cost function

$$K(x) = 1800 + 36x - 0.24x^2 + 0.002x^3$$

where x is the quantity produced (in hundreds) and K is in thousands of Euro.

Tasks:

a) Determine the fixed costs.
b) Calculate the total cost, average cost, and marginal cost for producing 60 (hundred) units.
c) Find the short-term lower limit price (kurzfristige Preisuntergrenze).
d) Find the long-term lower limit price (langfristige Preisuntergrenze).
e) The company receives an offer to purchase 20 (hundred) units at a price of 32 (thousand Euro per hundred). The company currently produces 50 (hundred) units. Should they accept this order? Justify your answer.

Problem 12: Graphical Analysis (xxx)

For the cost function $K(x) = 800 + 16x + 0.04x^2$:

a) Sketch the graphs of $k_v(x)$ and $\bar{K}(x)$ on the same axes for $x \in [10, 200]$.
b) Mark the short-term and long-term lower limit prices on your graph.
c) Shade the region representing the price range where short-term production is advisable but not long-term sustainable.
d) At approximately what quantity do the two curves come closest together? What does this tell us?