

Tasks 08-01 - Compound Interest & Geometric Sequences

Section 08: Financial Mathematics

Problem 1: Geometric Sequence Terms (x)

Find the specified term for each geometric sequence:

- a) $a_1 = 3, r = 2$: Find a_7
- b) $a_1 = 100, r = 0.5$: Find a_6
- c) $a_1 = 5, r = -3$: Find a_5
- d) 2, 6, 18, 54, ...: Find a_8

Problem 2: Geometric Series Sums (x)

Find the sum of each geometric series:

- a) $1 + 3 + 9 + 27 + \dots$ (first 8 terms)
- b) $64 + 32 + 16 + 8 + \dots$ (first 10 terms)
- c) $\sum_{k=0}^5 4 \cdot 2^k$
- d) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ (infinite sum)

Problem 3: Basic Compound Interest (x)

Calculate the future value for each investment:

- a) 5,000 invested at 4% annual interest for 10 years (annual compounding)
- b) 2,500 invested at 6% annual interest for 8 years (annual compounding)
- c) 10,000 invested at 3.5% annual interest for 15 years (annual compounding)

Problem 4: Multiple Compounding Periods (xx)

Calculate the future value with the given compounding frequency:

- a) 3,000 at 5% for 6 years, compounded semi-annually
- b) 8,000 at 4.8% for 4 years, compounded quarterly
- c) 1,500 at 6% for 3 years, compounded monthly
- d) 5,000 at 8% for 2 years, compounded daily

Problem 5: Effective Annual Rate (xx)

Find the effective annual rate (EAR) for each nominal rate:

- a) 6% compounded semi-annually
- b) 8% compounded quarterly
- c) 5% compounded monthly
- d) 4% compounded daily

Problem 6: Present Value (xx)

Find the present value (how much to invest today):

- a) To have 20,000 in 10 years at 5% annual interest
- b) To have 50,000 in 15 years at 4% annual interest
- c) To have 8,000 in 5 years at 6% compounded monthly
- d) To have 100,000 in 20 years at 7% compounded quarterly

Problem 7: Comparing Investments (xx)

Which investment offers a better return? Justify your answer.

- a) Bank A: 5.8% compounded monthly OR Bank B: 5.9% compounded annually
- b) Investment X: 7.2% compounded quarterly OR Investment Y: 7.0% compounded daily
- c) Option 1: 4.5% compounded semi-annually OR Option 2: 4.4% compounded monthly

Problem 8: Rule of 72 (x)

Use the Rule of 72 to estimate doubling time, then verify with exact calculation:

- a) 6% annual interest
- b) 9% annual interest
- c) 12% annual interest

Problem 9: Continuous Compounding (xx)

Calculate the future value with continuous compounding:

- a) 4,000 at 5% for 10 years
- b) 7,500 at 3.5% for 8 years
- c) 2,000 at 8% for 5 years

Then find the effective annual rate for each.

Problem 10: Business Application (xxx)

A company invests 250,000 Euro from profits into a bond fund.

- a) If the fund earns 5.5% compounded quarterly, what will it be worth in 7 years?
- b) What is the effective annual rate of return?
- c) How long until the investment doubles? (Use Rule of 72 and exact)
- d) If they need 400,000 Euro in 8 years, what annual rate (compounded annually) do they need?

Problem 11: Inflation and Real Returns (xxx)

An investment earns 7% nominal return while inflation is 2.5%.

- a) Calculate the approximate real return using the simple formula.
- b) Calculate the exact real return using the Fisher equation.

c) If you invest 10,000 today, what is the real purchasing power after 5 years?

Problem 12: Mixed Problem (xxxx)

You have 15,000 Euro to invest. You're comparing three options:

- Option A: 4.8% compounded monthly for 10 years
 - Option B: 5.0% compounded annually for 10 years
 - Option C: 4.6% compounded continuously for 10 years
- a) Calculate the future value for each option.
 - b) Calculate the effective annual rate for each option.
 - c) Which option gives the best return?
 - d) How much more does the best option earn compared to the worst?