Tasks 07-03 - Combinatorics

Section 07: Probability & Statistics

Problem 1: Fundamental Counting Principle (x)

a) A restaurant offers 4 appetizers, 6 main courses, and 3 desserts. How many different three-course meals can be ordered?

b) A password consists of 2 letters followed by 4 digits. How many different passwords are possible?

c) A product code has 3 letters followed by 2 digits. If letters and digits can repeat, how many codes are possible?

Problem 2: Factorials (x)

Calculate:

a) $5!$
b) $8!$
c) $\frac{10!}{8!}$
d) $\frac{12!}{10! \cdot 2!}$

Problem 3: Permutations (x)

a) In how many ways can 5 different books be arranged on a shelf?

b) How many different 3-letter "words" (not necessarily real words) can be formed from the letters A, B, C, D, E if no letter can be repeated?

c) A race has 8 runners. In how many ways can gold, silver, and bronze medals be awarded?

Problem 4: Combinations (x)

a) A committee of 3 people must be selected from a group of 10. How many different committees are possible?

b) A pizza shop offers 8 toppings. How many different 3-topping pizzas can be made?

c) From a deck of 52 cards, how many different 5-card hands can be dealt?

Problem 5: Permutations vs. Combinations (xx)

Determine whether each situation involves permutations or combinations, then solve:

a) Selecting 4 students from a class of 20 to represent the class at a conference.

b) Arranging 4 students from a class of 20 in a line for a photo (positions matter).

c) Choosing 3 flavors of ice cream from 12 available flavors.

d) Ranking your top 3 favorite movies from a list of 10.

Problem 6: Permutations with Repetition (xx)

a) How many different "words" can be formed using all the letters in MISSISSIPPI?

b) How many different arrangements are possible for the letters in STATISTICS?

c) A shelf has 3 identical math books, 4 identical physics books, and 2 identical chemistry books. How many ways can they be arranged?

Problem 7: Combinations with Conditions (xx)

From a group of 8 men and 6 women:

a) How many committees of 5 can be formed?

b) How many committees of 5 with exactly 3 men and 2 women can be formed?

c) How many committees of 5 with at least 3 women can be formed?

Problem 8: Probability with Counting (xx)

A standard deck has 52 cards (13 cards in each of 4 suits).

a) If 5 cards are dealt, what is the probability of getting all hearts?

b) What is the probability of getting exactly 3 aces in a 5-card hand?

c) What is the probability of getting a "full house" (3 of one rank, 2 of another)?

Problem 9: Business Applications (xx)

a) A company needs to select 4 employees from 12 for a project team. How many ways can this be done?

b) A manager must assign 4 different tasks to 4 of her 10 employees (one task per person). How many ways can this be done?

c) A product code consists of 2 letters (A-Z) followed by 3 digits (0-9). If repetition is allowed, how many codes are possible? If repetition is NOT allowed?

Problem 10: Pascal's Triangle and Binomial Coefficients (xx)

a) Write out the first 6 rows of Pascal's triangle.

b) Use Pascal's triangle to find $\binom{5}{2}$ and $\binom{5}{3}$.

c) Verify that $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$ using $n = 4$, $r = 2$.

d) Expand $(x + y)^4$ using binomial coefficients.

Problem 11: Complex Counting Problems (xxx)

a) How many 4-digit numbers can be formed using digits 1, 2, 3, 4, 5, 6 if:

- Repetition is allowed?
- Repetition is not allowed?
- The number must be even (no repetition)?

b) A club with 15 members needs to elect a president, vice president, secretary, and treasurer. No one can hold more than one position. How many ways can this be done?

c) From 8 different books, how many ways can you select 3 to give as gifts to 3 different friends (each friend gets one book)?

Problem 12: Comprehensive Problem (xxxx)

A small business has 5 managers and 10 regular employees.

a) How many ways can a committee of 4 be formed from all employees?

b) How many ways can a committee of 4 be formed with at least 2 managers?

c) If the committee must have a chair, vice-chair, secretary, and member, and the chair must be a manager, how many ways can the committee be formed?

d) What is the probability that a randomly selected 4-person committee has exactly 1 manager?