Tasks 06-05 - Economic Applications & Integration by Parts

Section 06: Integral Calculus

Problem 1: Consumer Surplus (x)

For each demand function and equilibrium price, calculate the consumer surplus:

a) $D(q) = 100 - 2q$, equilibrium at $q^* = 20$, $p^* = 60$

b) $D(q) = 80 - q$, equilibrium at $q^* = 30$, $p^* = 50$

c) $D(q) = 200 - 5q$, equilibrium at $q^* = 24$, $p^* = 80$

d) $D(q) = 150 - 3q$, equilibrium at $q^* = 25$, $p^* = 75$

Problem 2: Producer Surplus (x)

For each supply function and equilibrium price, calculate the producer surplus:

a) $S(q) = 10 + q$, equilibrium at $q^* = 20$, $p^* = 30$

b) $S(q) = 5 + 2q$, equilibrium at $q^* = 15$, $p^* = 35$

c) $S(q) = 20 + 0.5q$, equilibrium at $q^* = 40$, $p^* = 40$

d) $S(q) = 15 + 3q$, equilibrium at $q^* = 10$, $p^* = 45$

Problem 3: Complete Market Analysis (xx)

For each market, find equilibrium, then calculate CS, PS, and total surplus:

a) $D(q) = 120 - 4q$ and $S(q) = 20 + 2q$

b) $D(q) = 90 - 3q$ and $S(q) = 30 + q$

c) $D(q) = 200 - 2q$ and $S(q) = 40 + 2q$

Problem 4: Average Value (xx)

Find the average value of each function on the given interval:

a) $f(x) = 3x^2$ on $[0, 2]$

b) $f(x) = x^3 - x$ on $[0, 2]$

c) $f(x) = e^x$ on $[0, 3]$

d) $f(x) = \frac{1}{x}$ on $[1, e]$

e) $f(x) = 4 - x^2$ on $[-2, 2]$

Problem 5: Revenue and Cost Accumulation (xx)

a) A company's marginal revenue is $MR(x) = 150 - 3x$ EUR/unit. Find the additional revenue from increasing production from 20 to 40 units.

b) Marginal cost is $MC(x) = 20 + 0.5x$ EUR/unit. Find the total cost of producing the first 50 units (assuming no fixed costs).

c) If $MR(x) = 100 - 2x$ and $MC(x) = 10 + x$, find the profit-maximizing quantity and the total profit earned up to that quantity (starting from 0).

Problem 6: Business Applications (xx)

a) A factory's production rate is $P(t) = 120 - 4t$ units per hour, where t is hours into the shift. Find the average production rate over an 8-hour shift.

b) Daily sales revenue follows $R(t) = 500 + 100\sin(\frac{\pi t}{12})$ EUR, where t is hours after midnight. Find the total revenue from 6 AM to 6 PM (12 hours).

c) A machine depreciates at rate $V'(t) = -5000e^{-0.2t}$ EUR per year. Find the total depreciation over the first 5 years.

Problem 7: Profit Rate Analysis (xxx)

A startup's monthly profit rate is $P'(t) = 12t - t^2$ thousand EUR, where t is months after launch.

a) During which months is the company profitable (positive profit rate)?

b) Find the total profit accumulated from launch until the profit rate first becomes zero.

c) Find the month when the instantaneous profit rate is highest, and what is that maximum rate?

d) Calculate the average monthly profit rate over the first 12 months.

Problem 8: Basic Integration by Parts (x)

Use integration by parts to evaluate the following integrals. Remember to verify your answers by differentiation.

a) $\int x \cdot e^x \, dx$

b) $\int 2x \cdot e^x \, dx$

c) $\int x \cdot e^{-x} \, dx$

d) $\int (x+1) \cdot e^x \, dx$

e) $\int (x-2) \cdot e^x \, dx$

f) $\int 5x \cdot e^{2x} \, dx$

Problem 9: Integration with Logarithms (x)

Use the LIATE rule to determine which function should be u, then integrate.

a) $\int x \cdot \ln(x)\,dx$

b) $\int x^2 \cdot \ln(x)\,dx$

c) $\int \ln(x)\,dx$ (Hint: write as $\int 1 \cdot \ln(x)\,dx$)

d) $\int x^3 \cdot \ln(x)\,dx$

Problem 10: Repeated Integration by Parts (xx)

These integrals require applying integration by parts twice.

a) $\int x^2 \cdot e^x\,dx$

b) $\int x^2 \cdot e^{-x}\,dx$

c) $\int x^2 \cdot e^{2x}\,dx$

d) $\int (x^2 + x) \cdot e^x\,dx$

Problem 11: Definite Integrals by Parts (xx)

Evaluate the following definite integrals.

a) $\int_0^1 x \cdot e^x\,dx$

b) $\int_0^2 x \cdot e^{-x}\,dx$

c) $\int_1^e x \cdot \ln(x)\,dx$

d) $\int_0^1 x^2 \cdot e^x\,dx$

e) $\int_0^2 (x+1) \cdot e^x\,dx$

Problem 12: Exam-Style Problem - 2025 Format (xx)

Consider the function $f(x) = (x+1)e^x - 1$.

a) Find $\int f(x)\,dx$

b) Evaluate $\int_0^2 f(x)\,dx$

c) Find where $f(x) = 0$ and interpret what this means for the integral.

d) Sketch the graph of $f(x)$ for $-3 \leq x \leq 2$ and shade the region whose area is computed in part (b).

Problem 13: Exam-Style Problem - 2023 Format (xxx)

Evaluate $\int_0^1 x^2 \cdot e^{-x}\,dx$.

Show all steps clearly, including:

a) Setting up integration by parts (identify u and dv)
b) The first application of integration by parts
c) The second application of integration by parts
d) Combining results and evaluating the definite integral
e) Express your answer in exact form and as a decimal approximation

Problem 14: Business Application - Accumulated Profit (xx)

A company's marginal profit function (rate of profit in thousands of euros per month) is given by:

$$MP(t) = P'(t) = (20 - 2t) \cdot e^{-0.1t}$$

where t is months since product launch.

a) Find the accumulated profit function $P(t)$, given that at launch $(t = 0)$, the initial investment creates a loss of €50,000, so $P(0) = -50$.

b) Calculate the total profit from month 0 to month 12.

c) At what time does the marginal profit equal zero? What does this mean for the business?

d) What is the maximum accumulated profit, and when does it occur?