# Tasks 06-04 - Area Between Curves

## Section 06: Integral Calculus

## Problem 1: Finding Intersection Points (x)

Find the x-coordinates where the following pairs of functions intersect:

a) $f(x) = x^2$ and $g(x) = 4$

b) $f(x) = x + 2$ and $g(x) = x^2$

c) $f(x) = 6 - x$ and $g(x) = x$

d) $f(x) = x^2 - 1$ and $g(x) = 3 - x^2$

e) $f(x) = x^3$ and $g(x) = x$

## Problem 2: Determining Upper and Lower Functions (x)

For each interval, determine which function is on top (greater):

a) $f(x) = 4$ and $g(x) = x^2$ on $[-2, 2]$

b) $f(x) = x + 2$ and $g(x) = x^2$ on $[-1, 2]$

c) $f(x) = x$ and $g(x) = x^3$ on $[0, 1]$

d) $f(x) = \sqrt{x}$ and $g(x) = x^2$ on $[0, 1]$

## Problem 3: Basic Area Between Curves (xx)

Find the area enclosed between the given curves:

a) $f(x) = x + 1$ and $g(x) = x^2 - 1$ from $x = -1$ to $x = 2$

b) $f(x) = 4 - x^2$ and $g(x) = 0$ (the x-axis)

c) $f(x) = x^2$ and $g(x) = x$ from $x = 0$ to $x = 1$

d) $f(x) = 6 - x^2$ and $g(x) = x$

## Problem 4: Area with Multiple Regions (xxx)

Find the total area between the curves. Note: the curves may cross, creating multiple regions.

a) $f(x) = x$ and $g(x) = x^3$ from $x = -1$ to $x = 1$

b) $f(x) = \sin(x)$ and $g(x) = 0$ from $x = 0$ to $x = 2\pi$

c) $f(x) = x^2 - 4$ and $g(x) = 4 - x^2$

## Problem 5: Supply and Demand Curves (xx)

For a product, the demand curve is $D(q) = 100 - 2q$ and the supply curve is $S(q) = 20 + 3q$, where $q$ is quantity (in hundreds) and prices are in euros.

a) Find the equilibrium quantity $q^*$ and price $p^*$.

b) Set up the integral for the area between the demand and supply curves from $q = 0$ to $q^*$.

c) Calculate this area. (This represents total surplus, which we'll study in depth next session.)

## Problem 6: Cost and Revenue Functions (xx)

A company's marginal revenue is $MR(x) = 200 - 4x$ and marginal cost is $MC(x) = 40 + 2x$, where $x$ is quantity in thousands.

a) Find where marginal revenue equals marginal cost.

b) Calculate the area between the MR and MC curves from $x = 0$ to this break-even point.

c) Interpret this area in business terms.

## Problem 7: Bounded Regions (xx)

Find the area of the region bounded by:

a) $y = x^2$, $y = 0$, $x = 1$, and $x = 3$

b) $y = \sqrt{x}$, $y = 0$, and $x = 4$

c) $y = e^x$, $y = 1$, $x = 0$, and $x = 2$

d) $y = x^2$ and $y = 2x$ (find intersection first, then calculate enclosed area)

## Problem 8: Application - Profit Over Time (xxx)

A startup's revenue rate is $R'(t) = 50 + 10t$ thousand euros per month and cost rate is $C'(t) = 70 - 2t$ thousand euros per month, where $t$ is time in months.

a) At what time does revenue rate equal cost rate (break-even point in rates)?

b) Set up and evaluate the integral for total loss during the period when costs exceed revenues.

c) Set up and evaluate the integral for total profit during the period from break-even to $t = 10$ months.

d) What is the net profit/loss over the first 10 months?