Tasks 06-03 - Area Problems & Basic Applications

Section 06: Integral Calculus

Problem 1: Area Under Curves (x)

Find the area under each curve on the given interval (all functions are non-negative on these intervals):

a) $f(x) = 4x$ from $x = 0$ to $x = 3$

b) $f(x) = x^2 + 1$ from $x = 0$ to $x = 2$

c) $f(x) = 3x^2 - 2x + 5$ from $x = 1$ to $x = 4$

d) $f(x) = \sqrt{x}$ from $x = 1$ to $x = 9$

Problem 2: Total Area with Sign Changes (xx)

For each function, find the total area (not signed area) between the curve and the x-axis on the given interval:

a) $f(x) = x$ from $x = -3$ to $x = 4$

b) $f(x) = x^2 - 4$ from $x = 0$ to $x = 4$

c) $f(x) = x - 1$ from $x = -1$ to $x = 3$

d) $f(x) = x^2 - 2x$ from $x = -1$ to $x = 3$

Problem 3: Exponential Integrals (x)

Evaluate the following integrals:

a) $\int e^{2x}\,dx$

b) $\int 3e^{-4x}\,dx$

c) $\int_0^1 e^{3x}\,dx$

d) $\int_0^2 (e^x + 1)\,dx$

e) $\int_{-1}^1 e^{-x}\,dx$

f) $\int (2e^{5x} - e^{-x})\,dx$

Problem 4: Logarithmic Integrals (x)

Evaluate the following integrals:

a) $\int_1^e \frac{1}{x}\,dx$

b) $\int_1^4 \frac{2}{x}\,dx$

c) $\int_2^6 \frac{3}{x}\,dx$

d) $\int_1^e \frac{5}{x}\,dx$

e) $\int \left(\frac{1}{x} + x\right) dx$

f) $\int_1^{e^2} \frac{1}{x}\,dx$

Problem 5: Mixed Integration (xx)

Evaluate the following integrals:

a) $\int_0^4 (x^2 + e^x)\,dx$

b) $\int_1^e \left(x + \frac{1}{x}\right) dx$

c) $\int_0^2 (3e^{2x} - 4x)\,dx$

d) $\int_1^4 \left(\sqrt{x} + \frac{2}{x}\right) dx$

Problem 6: Business Rate Problems (xx)

a) A company's marginal profit is $MP(x) = 100 - 0.5x$ dollars per unit. Find the total profit from producing units 50 to 150.

b) The rate of oil production from a well is $R(t) = 500e^{-0.1t}$ barrels per day. Find the total production during the first 10 days.

c) A factory's production rate is $P(t) = 80 + 20t - t^2$ units per hour. Find the total production during an 8-hour shift.

d) A company's revenue rate is $R'(t) = 2000 - 50t$ euros per day. Find the total revenue during days 1 through 20.

Problem 7: Area and Economics Application (xxx)

A new product's daily sales rate (units per day) follows the pattern:

$$S(t) = 200e^{-0.05t}$$

where t is days after launch.

a) Find the total sales during the first month (30 days).

b) Find the total sales during the second month (days 30 to 60).

c) What percentage of first-month sales are achieved in the second month?

d) After how many days will total cumulative sales reach 3,000 units?

e) What is the average daily sales rate during the first 30 days?

Problem 8: Comprehensive Problem (xxx)

A company launches a subscription service with the following characteristics:

- New subscriber rate: $N(t) = 500 - 10t$ subscribers per month (for $t \leq 50$)
- Churn rate (subscribers leaving): $C(t) = 50 + 5t$ subscribers per month
- Monthly revenue per subscriber: €15

where t is months since launch.

Part A: Subscriber Analysis

a) Find the net subscriber growth rate function $G(t) = N(t) - C(t)$.

b) During which month does net growth become zero (break-even growth)?

c) Calculate the net new subscribers gained during the first year (months 0-12).

Part B: Revenue Analysis

d) If the company starts with 1,000 subscribers, write a function for total subscribers $S(t)$ at time t.

e) Find the total subscriber-months during the first year. (Hint: This is $\int_0^{12} S(t)\,dt$)

f) Calculate the total revenue for the first year.

Part C: Strategic Decisions

g) At what month should the company expect maximum total subscribers?

h) If marketing costs €5 per new subscriber acquired, what are total marketing costs for the first year?