Tasks 06-02 - Definite Integrals & The Fundamental Theorem

Section 06: Integral Calculus

Problem 1: Basic Definite Integrals (x)

Evaluate the following definite integrals using the Fundamental Theorem of Calculus:

a) $\int_0^3 4x\,dx$

b) $\int_1^2 x^2\,dx$

c) $\int_0^4 (2x+3)\,dx$

d) $\int_{-1}^2 x^3\,dx$

e) $\int_1^5 6\,dx$

f) $\int_0^1 (x^2 - x + 1)\,dx$

Problem 2: Polynomial Integrals (x)

Evaluate these definite integrals:

a) $\int_0^2 (3x^2 + 2x - 1)\,dx$

b) $\int_{-2}^3 (x^2 - 4)\,dx$

c) $\int_1^4 (x^3 - x)\,dx$

d) $\int_0^3 (4x^3 - 6x^2 + 2x)\,dx$

Problem 3: Integrals with Roots and Powers (xx)

Evaluate the following:

a) $\int_1^4 \sqrt{x}\,dx$

b) $\int_1^8 x^{2/3}\,dx$

c) $\int_1^4 \frac{3}{\sqrt{x}}\,dx$

d) $\int_1^2 \frac{1}{x^2}\,dx$

e) $\int_4^9 \frac{x+1}{\sqrt{x}}\,dx$

Problem 4: Properties of Definite Integrals (xx)

Use the properties of definite integrals to answer the following:

a) Given that $\int_0^5 f(x)\,dx = 12$, find $\int_5^0 f(x)\,dx$.

b) Given that $\int_1^4 g(x)\,dx = 8$ and $\int_1^4 h(x)\,dx = 3$, find $\int_1^4 [2g(x) - 3h(x)]\,dx$.

c) Given that $\int_0^3 f(x)\,dx = 7$ and $\int_3^6 f(x)\,dx = 5$, find $\int_0^6 f(x)\,dx$.

d) Given that $\int_0^8 f(x)\,dx = 20$ and $\int_0^3 f(x)\,dx = 9$, find $\int_3^8 f(x)\,dx$.

e) Evaluate $\int_5^5 (x^3 + 2x^2 - 7x + 100)\,dx$ without computing any antiderivatives.

Problem 5: Signed Area Problems (xx)

For each function, calculate:

- (i) The signed area (definite integral)
- (ii) The total (unsigned) area between the curve and the x-axis

a) $f(x) = x$ from $x = -3$ to $x = 2$

b) $f(x) = x^2 - 4$ from $x = 0$ to $x = 3$

c) $f(x) = x - 2$ from $x = 0$ to $x = 4$

Problem 6: Net Change Applications (xx)

Apply the Net Change Theorem to solve these problems:

a) A tank is being filled with water at a rate of $r(t) = 50 - 2t$ liters per minute. How much water enters the tank during the first 10 minutes?

b) The marginal cost of producing x units is $MC(x) = 0.04x + 5$ dollars. Find the increase in cost when production increases from 100 to 200 units.

c) A population of bacteria grows at a rate of $P'(t) = 100e^{0.1t}$ bacteria per hour. What is the total increase in population during the first 5 hours? (Leave your answer in terms of e.)

Problem 7: Business Applications (xxx)

A company's marginal revenue and marginal cost functions are:

$$MR(x) = 120 - 0.4x$$

$$MC(x) = 40 + 0.2x$$

where x is the number of units produced and sold.

a) Find the total revenue from selling the first 100 units, given that $R(0) = 0$.

b) Find the total cost of producing the first 100 units, given that fixed costs are $C(0) = 2000$.

c) Find the total profit from producing and selling the first 100 units.

d) At what production level does marginal revenue equal marginal cost? What does this represent?

e) Calculate the additional profit gained by increasing production from 100 units to the level found in part (d).

Problem 8: Comprehensive Problem (xxx)

A company manufactures smart watches. Their production analysis reveals:

- Marginal cost: $MC(x) = 50 + 0.02x$ dollars per watch
- Fixed costs: $F = \$10,000$ per month
- Selling price: $p = \$150$ per watch
- Maximum production capacity: 5,000 watches per month

Part A: Cost Analysis

a) Find the total cost function $C(x)$.

b) Calculate the total cost of producing 2,000 watches.

c) Calculate the average cost per watch when producing 2,000 watches.

Part B: Revenue and Profit

d) Find the revenue function $R(x)$ and profit function $P(x)$.

e) Calculate the profit when producing and selling 2,000 watches.

f) How many watches must be produced to break even (profit = 0)?

Part C: Optimization

g) At what production level is profit maximized? What is the maximum profit?

h) If the company is currently producing 3,000 watches, should they increase or decrease production? Justify using marginal analysis.