Tasks 06-01 - Antiderivatives & Indefinite Integrals

Section 06: Integral Calculus

Problem 1: Basic Power Rule (x)

Evaluate the following indefinite integrals:

a) $\int x^5 \, dx$

b) $\int 3x^4 \, dx$

c) $\int (x^2 + x + 1) \, dx$

d) $\int (4x^3 - 2x^2 + 5) \, dx$

e) $\int 7 \, dx$

f) $\int (x^6 - x^3 + 2x) \, dx$

Problem 2: Negative and Fractional Powers (x)

Rewrite each expression using exponents, then integrate:

a) $\int \frac{1}{x^2} \, dx$

b) $\int \frac{3}{x^4} \, dx$

c) $\int \sqrt{x} \, dx$

d) $\int \frac{1}{\sqrt{x}} \, dx$

e) $\int (x^2 + \frac{1}{x^3}) \, dx$

f) $\int (\sqrt{x} - \frac{2}{x^2}) \, dx$

Problem 3: Initial Value Problems (x)

Find the function $f(x)$ that satisfies each condition:

a) $f'(x) = 3x^2$, $f(1) = 5$

b) $f'(x) = 2x + 4$, $f(0) = 3$

c) $f'(x) = 6x^2 - 4x + 1$, $f(2) = 10$

d) $f'(x) = x^3 - 1$, $f(-1) = 2$

Problem 4: Marginal Cost to Total Cost (xx)

A manufacturing company produces electronic components. The marginal cost function (cost of producing one additional unit) is given by:

$$MC(x) = C'(x) = 0.02x^2 - 2x + 50$$

where x is the number of units produced and cost is in euros.

a) Find the general antiderivative of $MC(x)$.

b) If the fixed costs (costs when $x = 0$) are €1,200, find the specific total cost function $C(x)$.

c) Calculate the total cost of producing 100 units.

d) Calculate the average cost per unit when producing 100 units.

e) Interpret the meaning of the constant of integration in this context.

Problem 5: Revenue and Profit Analysis (xx)

A company sells premium coffee subscriptions. Their marginal revenue function is:

$$MR(x) = R'(x) = 60 - 0.4x$$

where x is the number of subscriptions and revenue is in euros.

The marginal cost function is:

$$MC(x) = C'(x) = 20 + 0.2x$$

Fixed costs are €500.

a) Find the total revenue function $R(x)$, given that $R(0) = 0$.

b) Find the total cost function $C(x)$.

c) Find the profit function $P(x) = R(x) - C(x)$.

d) Find the marginal profit function $MP(x) = P'(x)$ and verify it equals $MR(x) - MC(x)$.

e) How many subscriptions maximize profit? What is the maximum profit?

Problem 6: Velocity and Position (xx)

A delivery drone takes off from a warehouse roof (10 meters above ground). Its vertical velocity is given by:

$$v(t) = 12 - 4t \quad \text{meters per second}$$

where t is time in seconds after takeoff.

a) Find the height function $h(t)$, using the initial condition $h(0) = 10$.

b) At what time does the drone reach its maximum height?

c) What is the maximum height reached?

d) When does the drone return to the height of the warehouse roof (10 meters)?

Problem 7: Compound Business Problem (xxx)

A startup company produces smart home devices. After analyzing production data, they model:

Marginal cost: $MC(x) = 0.003x^2 - 0.6x + 80$ euros per unit

Marginal revenue: $MR(x) = 120 - 0.2x$ euros per unit

where x is the number of units produced per month.

Fixed costs are €10,000 per month.

Part A: Function Determination

a) Find the total cost function $C(x)$.

b) Find the total revenue function $R(x)$ (assuming $R(0) = 0$).

c) Find the profit function $P(x)$.

Part B: Analysis

d) Calculate the profit or loss when producing 100 units.

e) Find the break-even point(s) where $P(x) = 0$.

f) Find the production level that maximizes profit.

g) Calculate the maximum monthly profit.

Part C: Interpretation

h) The company currently produces 80 units per month. Should they increase or decrease production? Justify your answer using marginal analysis.

Problem 8: Extended Application - Population Growth (xxx)

A city's population growth rate is modeled by:

$$P'(t) = 0.02t^2 - 0.3t + 2$$

where $P(t)$ is population in thousands and t is years since 2020.

In 2020 ($t = 0$), the population was 50,000 (or $P(0) = 50$ in thousands).

a) Find the population function $P(t)$.

b) What is the population in 2025 ($t = 5$)?

c) In which year was the growth rate at its minimum?

d) What is the minimum growth rate (in thousands per year)?

e) The city plans major infrastructure if population reaches 60,000. In what year will this occur?

f) Graph both $P'(t)$ and $P(t)$ for $0 \leq t \leq 15$ and interpret the relationship between them.