Practice Tasks - Session 05-07

Function Determination & Funktionsscharen

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Part 1: Quadratic Functions

Problem 1: Three Points (xx)

Find the quadratic function $f(x) = ax^2 + bx + c$ passing through the points $(0, 3)$, $(1, 6)$, and $(2, 11)$.

Problem 2: Vertex Form (x)

Find the equation of a parabola with vertex at $(-3, 5)$ that passes through the point $(0, -4)$.

Problem 3: Given Maximum (xx)

Find the quadratic function that has a maximum at $(2, 8)$ and passes through the origin.

Problem 4: Point and Slope Conditions (xx)

Find the quadratic function $f(x) = ax^2 + bx + c$ such that: - $f(1) = 4$ - $f'(1) = 3$ - $f(3) = 6$

Part 2: Cubic Functions

Problem 5: Four Points (xxx)

Find the cubic function $f(x) = ax^3 + bx^2 + cx + d$ passing through: $(0, 2)$, $(1, 1)$, $(-1, 3)$, and $(2, 6)$.

Problem 6: Two Extrema (xxxx)

Find the cubic function $f(x) = ax^3 + bx^2 + cx + d$ that has:

- A local maximum at $(0, 5)$
- A local minimum at $(2, 1)$

Problem 7: Inflection Point Condition (xxx)

Find the cubic function $f(x) = ax^3 + bx^2 + cx + d$ such that:

- $f(0) = 1$
- $f(1) = 2$
- $f'(0) = 3$

- Has an inflection point at $x = 2$

Problem 8: Mixed Conditions (xxxx)

Find the cubic function with:

- Passes through $(1, 4)$
- Has a local extremum at $x = 0$ with $f(0) = 2$
- Has $f'(2) = 6$

Part 3: Funktionsscharen (Function Families with Parameters)

Problem 9: Single Parameter (xx)

For the function family $f_a(x) = ax^2 - 4ax + 5$, find the value of a such that $f_a(3) = 2$.

Problem 10: Parameter with Extremum (xxx)

For $g_t(x) = x^3 - 3tx^2 + 4$, find the value(s) of t such that g_t has a local extremum at $x = 2$.

Problem 11: Two Parameters (xxxx)

Find values of a and b such that $f(x) = x^3 + ax^2 + bx$ has:

- A local maximum at $x = 1$ with $f(1) = 6$

Problem 12: Parameter with Two Conditions (xxxx)

For the function family $h_k(x) = kx^3 - 3kx + 2$, find k such that:

- h_k has a local extremum at $x = 1$
- $h_k(2) = 0$

Part 4: Business Applications

Problem 13: Cost from Marginal Cost (xx)

A company's marginal cost function is $MC(x) = C'(x) = 6x^2 - 8x + 15$, where x is thousands of units.

The fixed cost (cost when $x = 0$) is €2000.

Find the total cost function $C(x)$.

Problem 14: Revenue Function (xxx)

A company knows the following about its revenue function $R(x) = ax^3 + bx^2 + cx$ (in thousands):

- Revenue from selling 1000 units (x=1) is €50,000: $R(1) = 50$
- Revenue from selling 2000 units (x=2) is €140,000: $R(2) = 140$

- Marginal revenue at $x = 1$ is €70,000 per thousand units: $R'(1) = 70$

Find the revenue function.

Problem 15: Profit Optimization Design (xxxx)

An analyst wants to model a company's profit function as cubic: $P(x) = ax^3 + bx^2 + cx + d$ where x is production level in thousands.

Requirements:

- Fixed costs (losses when nothing is produced): $P(0) = -20$ (€20,000 loss)
- Break-even at 2000 units: $P(2) = 0$
- Maximum profit at 4000 units: Critical point at $x = 4$
- Profit at maximum is €60,000: $P(4) = 60$

Find the profit function.

Part 5: Advanced Funktionsscharen (Exam Practice)

These problems are typical exam questions. Master them!

Problem 17: Zeros with Discriminant (xxx)

For the function family $f_t(x) = x^2 + 2tx + t + 6$:

a) For which values of t does f_t have exactly two distinct zeros?

b) For which values of t does f_t have exactly one zero?

c) For which values of t does f_t have no zeros?

Problem 18: Extremum Location (xxx)

For $g_t(x) = x^3 - 3tx^2 + 12x$:

a) Find the value of t such that g_t has a local extremum at $x = 2$.

b) For that value of t, classify the extremum.

c) Find $g_t(2)$ for that value of t.

Problem 19: Inflection Point Parameter (xxx)

For $h_t(x) = x^3 + tx^2 - 9x + 5$:

a) Find t such that h_t has an inflection point at $x = 1$.

b) For that t, find the coordinates of the inflection point.

Problem 20: Function Value Condition (xx)

For $f_k(x) = kx^2 - 4x + k$:

a) Find k such that $f_k(1) = 3$.

b) For that k, find the vertex of the parabola.

c) Does f_k open upward or downward?

Problem 21: Two Zeros at Specific Points (xxxx)

For $g_t(x) = x^2 - tx + t - 3$:

Find t such that g_t has zeros at $x = 1$ and $x = 3$.

Problem 22: Maximum Value Parameter (xxx)

For $p_t(x) = -x^2 + 4x + t$:

a) What is the maximum value of p_t (in terms of t)?

b) Find t such that the maximum value is 10.

c) For that t, find the zeros of p_t.