Tasks: Graphical Calculus Mastery

Session 05-05 Practice Problems

EXAM: This type of problem appears on EVERY exam. Master these skills!

Problem 1: Polynomial Function (x)

Given the graph of $f(x)$ below, sketch the graph of $f'(x)$.

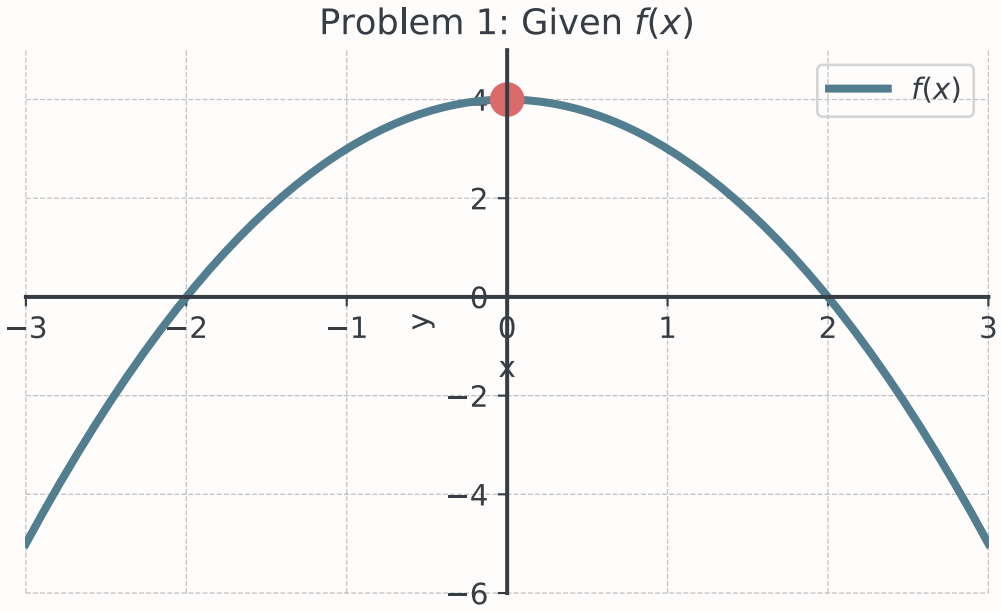

Figure 1: Sketch f'(x) for this parabola

Problem 2: Cubic Function (x)

Given the graph of $f(x)$, sketch $f'(x)$ and identify all critical points.

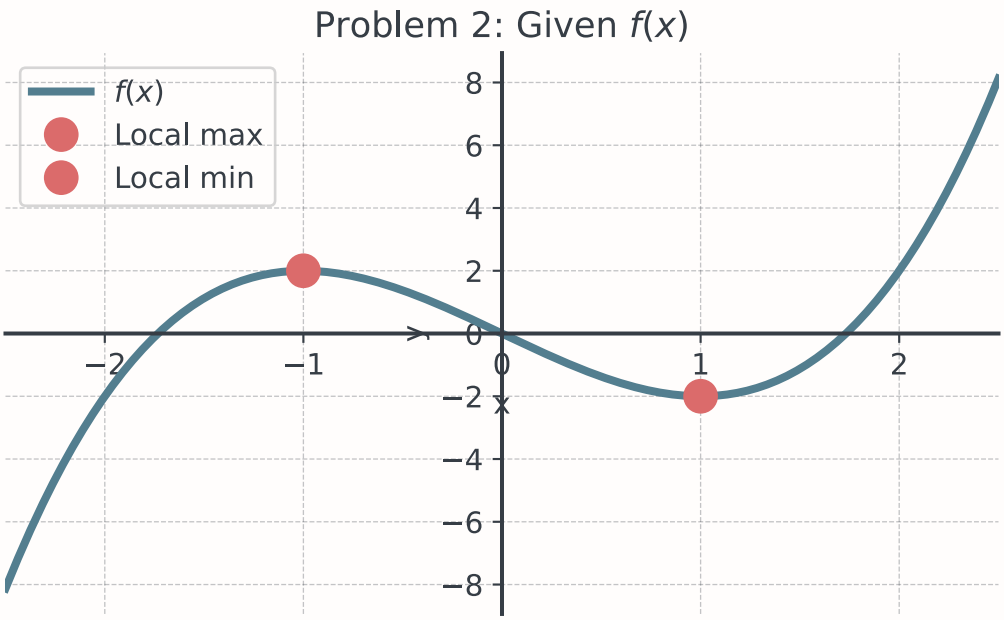

Figure 2: Sketch f'(x) for this cubic

Problem 3: Piecewise Linear Function (xx)

Sketch $f'(x)$ for the piecewise linear function shown. Where does $f'(x)$ not exist?

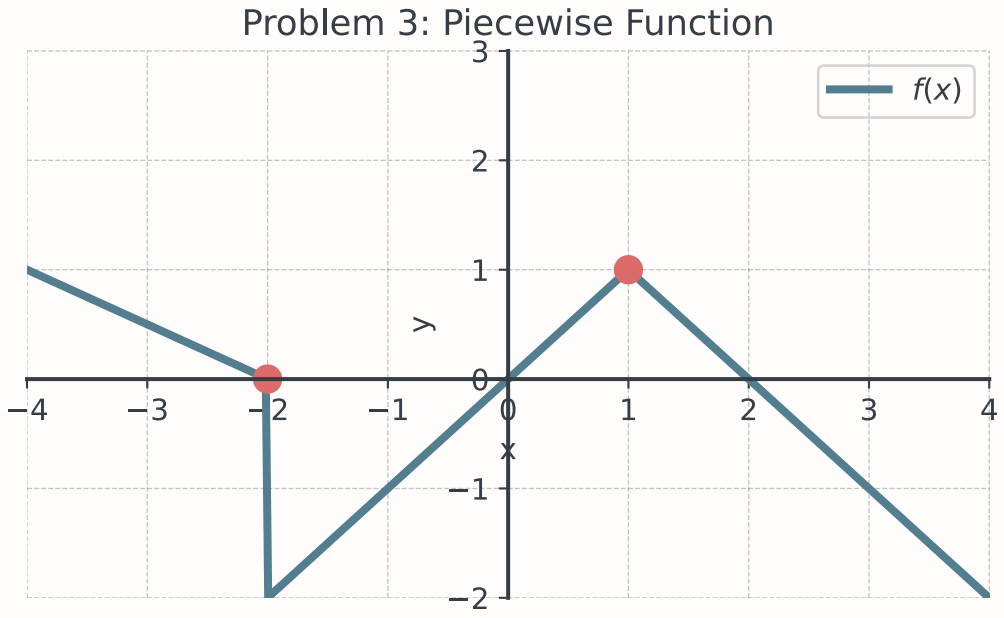

Figure 3: Piecewise linear function

Problem 4: Absolute Value Function (xx)

Sketch $f'(x)$ for $f(x) = |x - 2|$ on the interval $[-1, 5]$.

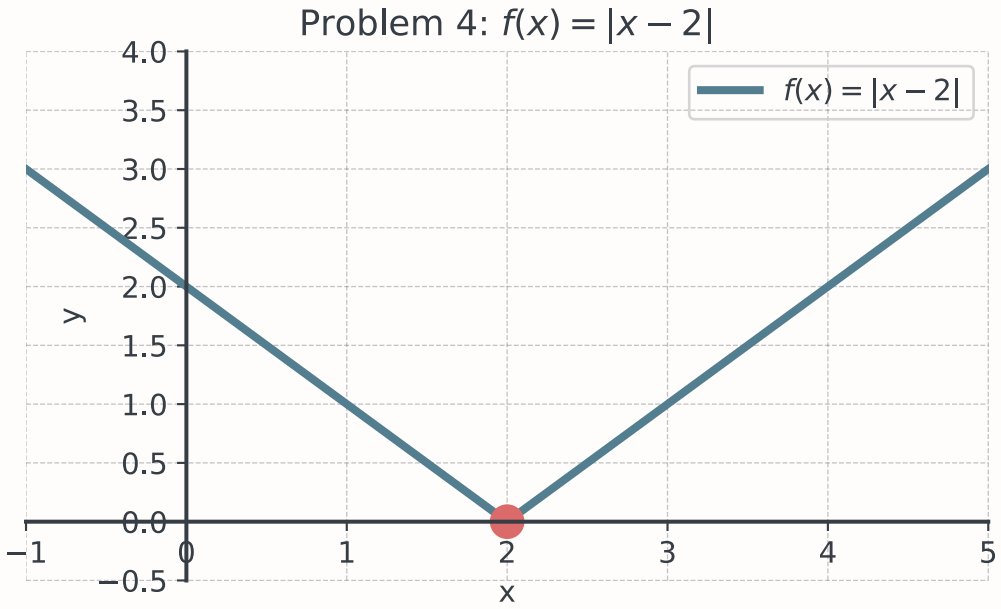

Figure 4: Absolute value function

Problem 5: Quartic with Multiple Extrema (xxx)

Sketch $f'(x)$ and $f''(x)$ for the quartic function shown.

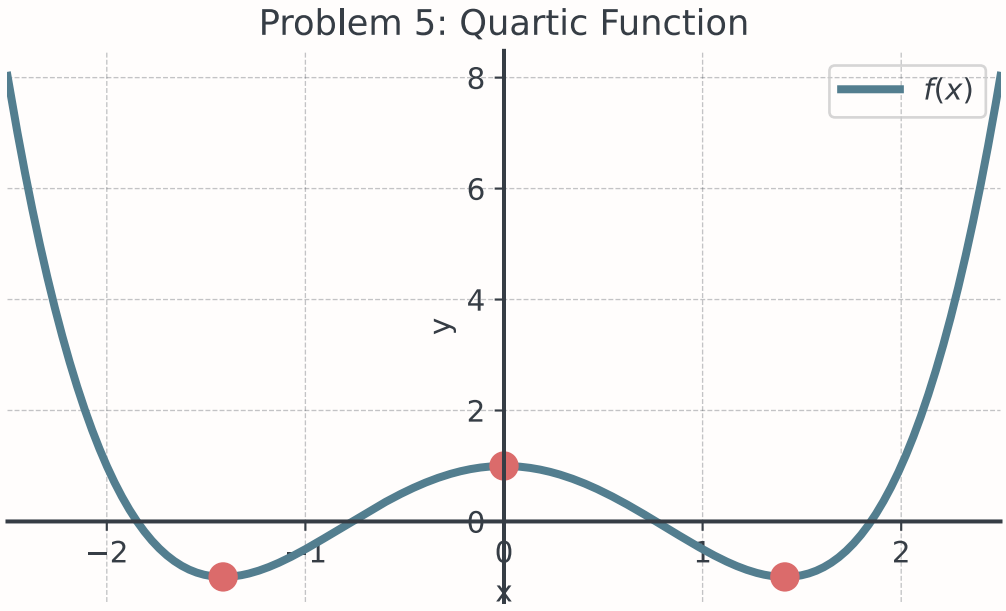

Figure 5: Quartic function with two minima and one maximum

Problems 6-10: Quick Sketches (x)

For each function graph below, sketch $f'(x)$. Identify critical points.

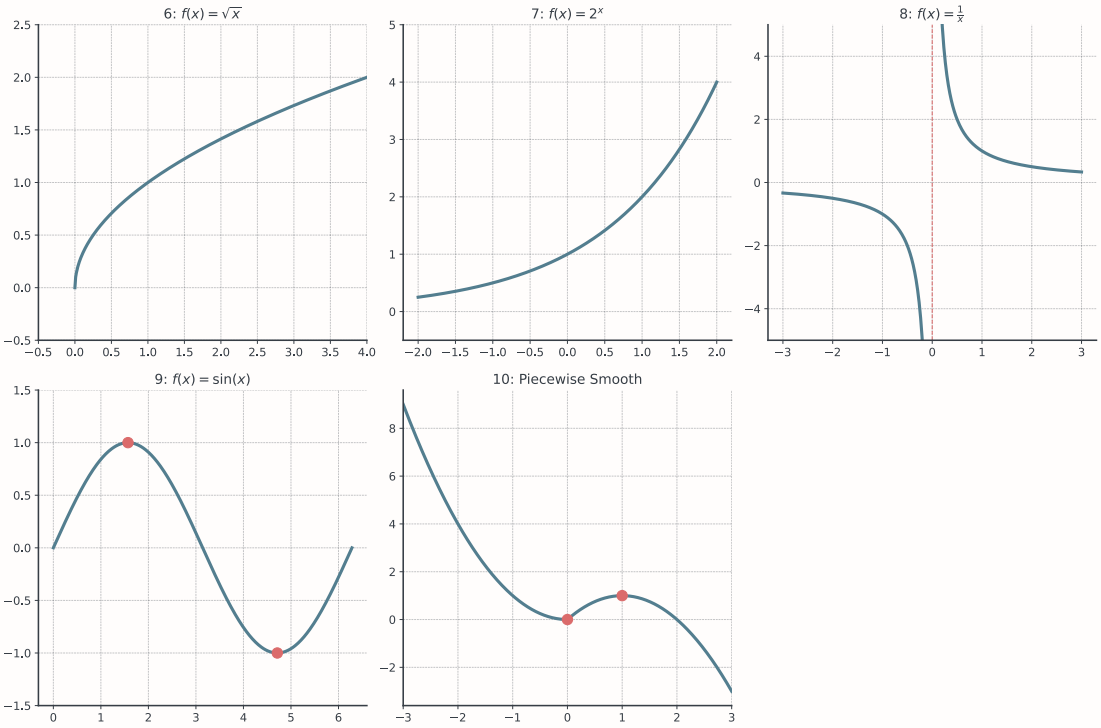

Figure 6: Five functions to practice

Problem 11: Linear Derivative (x)

Given the graph of $f'(x)$ below:

a) Where is $f(x)$ increasing? Decreasing?
b) Where does $f(x)$ have local extrema? Classify them.
c) Sketch a possible graph of $f(x)$.

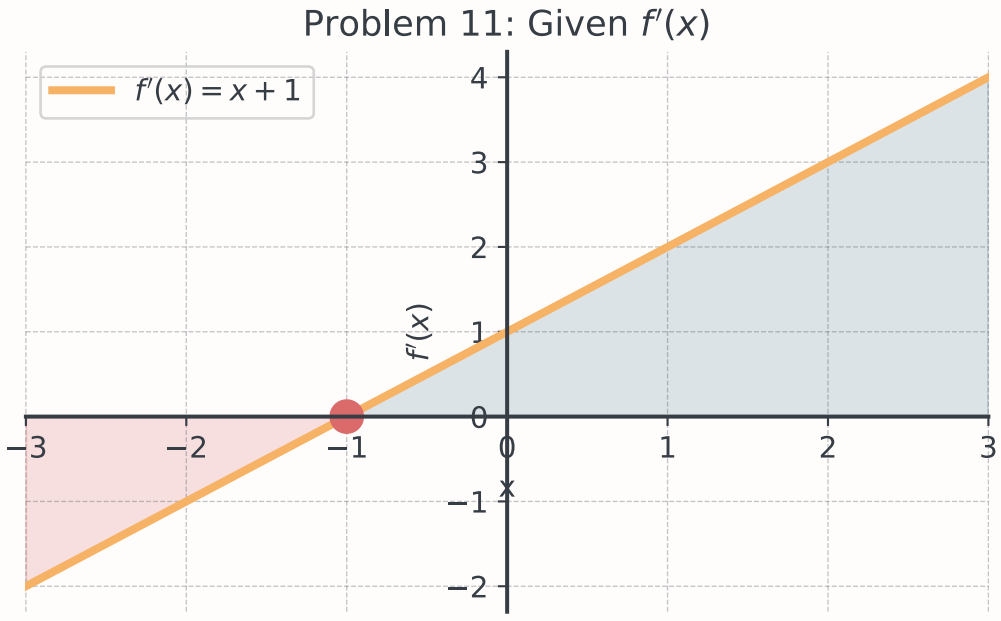

Figure 7: Linear derivative function

Problem 12: Quadratic Derivative (xx)

Given $f'(x)$ shown below:

a) Find all critical points of $f(x)$ and classify them.
b) Where is $f(x)$ concave up? Concave down?
c) Sketch $f(x)$.

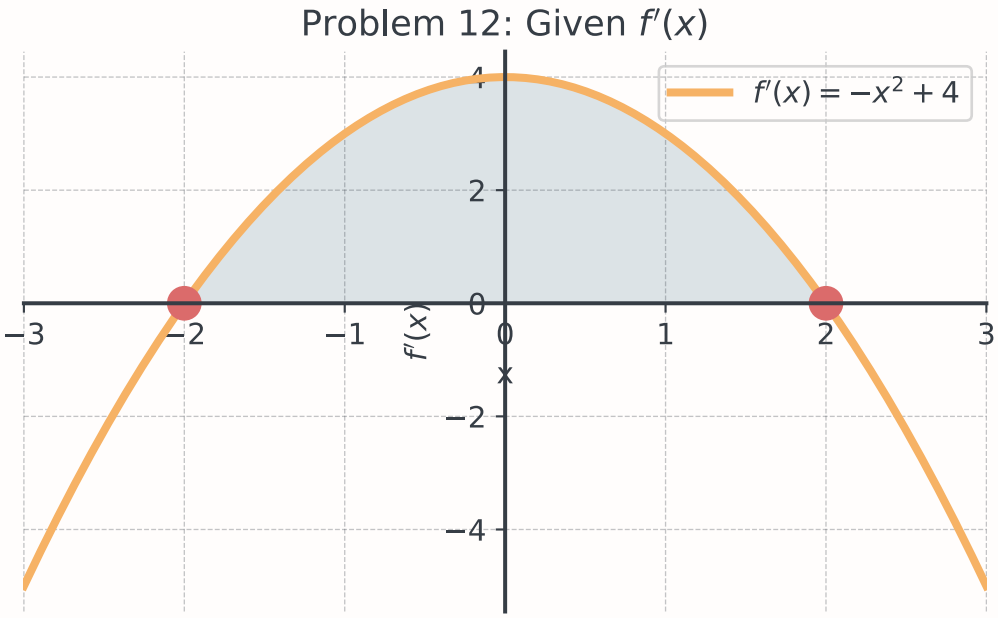

Figure 8: Parabolic derivative

Problem 13: Piecewise Constant Derivative (xx)

Given the step function $f'(x)$ below, sketch $f(x)$.

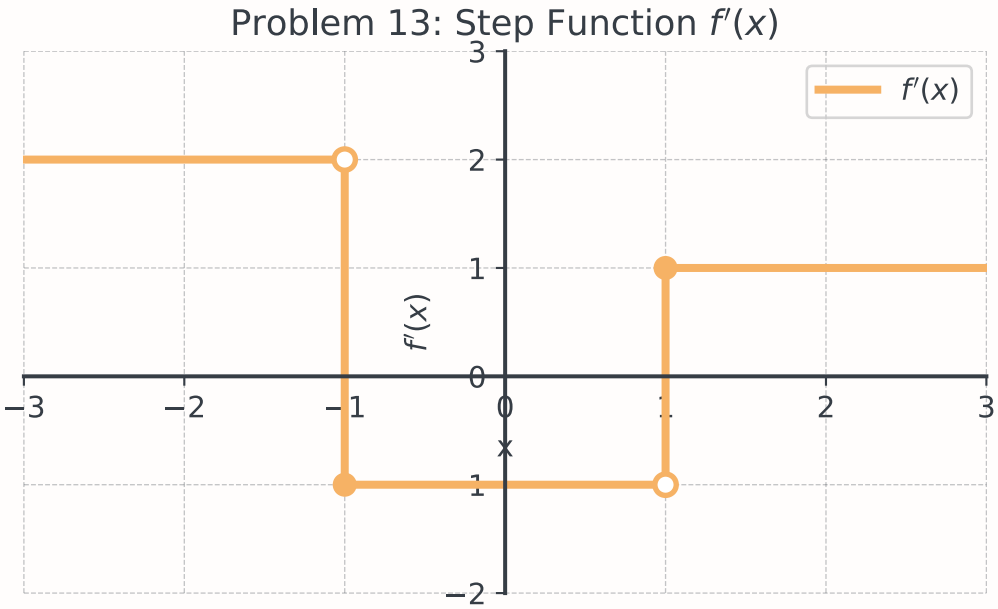

Figure 9: Step function derivative

Problems 14-20: Quick Analysis (xx)

For each derivative graph, answer: Where is f increasing? Where does f have local extrema?

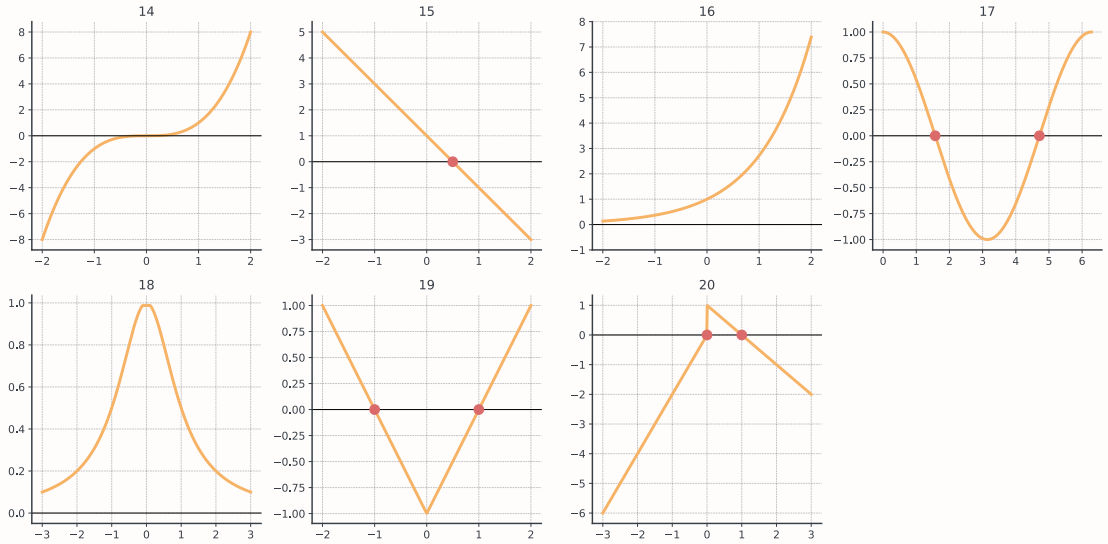

Figure 10: Seven derivative functions to analyze

Problem 21: Comprehensive Analysis (xxx)

Given $f(x) = x^4 - 4x^3 + 4x^2$:

a) Find $f'(x)$ and $f''(x)$.
b) Find all critical points and classify them.
c) Find all inflection points.
d) Determine intervals where f is increasing/decreasing and concave up/down.
e) Sketch the graphs of $f(x)$, $f'(x)$, and $f''(x)$ on the same set of axes.

Problem 22: Business Application (xxx)

A company's revenue over 10 months is modeled by:

$$R(t) = -t^3 + 9t^2 - 15t + 50$$

where t is months and R is in thousands €.

a) When is revenue increasing? Decreasing?
b) When does revenue reach local extrema? What is the revenue at these points?
c) When is the rate of revenue change accelerating? Decelerating?
d) Interpret all results in business terms.

Problem 23: Challenge Problem (xxxx)

Consider the piecewise function:

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 3 - x & \text{if } x \geq 1 \end{cases}$$

a) Is f continuous at $x = 1$?
b) Is f differentiable at $x = 1$?
c) Sketch $f(x)$ and $f'(x)$.
d) Classify $x = 1$ (corner, cusp, or smooth?).