Tasks: Chain Rule & Implicit Differentiation

Session 05-04 Practice Problems

Problem 1: Basic Chain Rule (x)

Differentiate the following functions using the chain rule:

a) $f(x) = (3x + 7)^6$

b) $g(x) = \sqrt{5x - 2}$

c) $h(x) = (x^2 - 4x + 1)^{10}$

d) $k(x) = \frac{1}{(2x+3)^4}$

Problem 2: Chain Rule with Product Rule (xx)

Differentiate the following functions:

a) $f(x) = x^3(2x - 1)^4$

b) $g(x) = (x^2 + 1)^2(3x - 5)^3$

c) $h(x) = \frac{x^2}{(x+1)^3}$

Problem 3: Simplifying Before Differentiating (x)

Differentiate by simplifying first:

a) $f(x) = \sqrt{(2x + 1)^3}$

b) $g(x) = \frac{1}{\sqrt{x^2+1}}$

Problem 4: Implicit Differentiation - Business Contexts (x)

Find the derivative for each business relationship:

a) A company's price p and quantity q satisfy a constant revenue constraint: $pq = 5000$. Find $\frac{dq}{dp}$.

b) Marketing spend M and sales S follow: $MS = 12000$. Find $\frac{dS}{dM}$.

c) Budget constraint: $30L + 50K = 9000$ where L = labor hours and K = capital units. Find $\frac{dK}{dL}$.

Problem 5: Related Rates - Customer Growth (xx)

A company's revenue R (in €1000) depends on its customer base C (in thousands):

$$R = 80\sqrt{C}$$

The company is gaining 2,000 new customers per month.

a) Find $\frac{dR}{dC}$ (marginal revenue per customer).

b) Find $\frac{dR}{dt}$ in terms of C and $\frac{dC}{dt}$.

c) How fast is revenue growing when $C = 100$ (100,000 customers)?

d) How fast is revenue growing when $C = 400$?

e) Explain why revenue growth slows as the customer base grows.

Problem 6: Related Rates - Production and Profit (xx)

A retail chain's annual profit P (in €1000) and number of stores n are related by:

$$P = 150\sqrt{n} - 4n$$

The company opens 3 new stores per year.

a) Find $\frac{dP}{dn}$ and interpret its meaning.

b) Find $\frac{dP}{dt}$ when $n = 25$ stores.

c) Find $\frac{dP}{dt}$ when $n = 100$ stores.

d) At how many stores does profit stop growing (i.e., $\frac{dP}{dn} = 0$)?

Problem 7: Related Rates - Market Share (xxx)

A company's profit P (in €1000) and market share m (as a percentage) are related by:

$$P = 800m - 15m^2$$

Market share is increasing at 1.5% per month.

a) Find $\frac{dP}{dt}$ in terms of m.

b) How fast is profit changing when $m = 10\%$?

c) How fast is profit changing when $m = 25\%$?

d) At what market share is profit maximized?

e) If the company currently has $m = 30\%$, should they continue trying to grow market share? Why or why not?