Tasks: Differentiation Rules & Tangent Lines

Session 05-03 Practice Problems

Problem 1: Basic Differentiation Rules (x)

Find the derivatives of the following functions using the appropriate rules:

a) $f(x) = 5x^4 - 3x^2 + 7x - 2$

b) $g(x) = 2\sqrt{x} + \frac{3}{x} - \frac{1}{x^3}$

c) $h(x) = \frac{1}{2}x^{-4} + 4x^{1/3}$

Problem 2: Product Rule Practice (x)

Differentiate the following functions using the product rule:

a) $f(x) = (3x^2 + 1)(x - 4)$

b) $g(x) = x^3(2x^2 - 5x + 1)$

c) Verify your answer to part (a) by first expanding, then differentiating.

Problem 3: Quotient Rule Applications (xx)

Find the derivatives using the quotient rule:

a) $f(x) = \frac{x^2+3}{x-1}$

b) $g(x) = \frac{2x-5}{3x+2}$

c) $h(x) = \frac{x^3-1}{x^2+1}$

Problem 4: Finding Tangent Lines (xx)

For each function, find the equation of the tangent line at the given point:

a) $f(x) = x^3 - 2x^2 + 1$ at $x = 2$

b) $g(x) = \frac{x+1}{x-1}$ at $x = 3$

c) $h(x) = x^2(x - 3)$ at the point where $x = 1$

Problem 5: Linear Approximation in Business (xx)

A company's profit function is:

$$P(x) = -0.2x^2 + 12x - 50$$

where x is the number of items sold (in thousands) and P is profit in thousands of dollars.

a) Find the profit when $x = 25$ thousand items.

b) Find the marginal profit function $P'(x)$ and evaluate it at $x = 25$.

c) Use linear approximation to estimate the profit when $x = 26$ thousand items.

d) Calculate the actual profit at $x = 26$ and compare with your estimate. What is the error?

e) Interpret the marginal profit at $x = 25$ in business terms.

Problem 6: Optimization with Marginal Analysis (xxx)

A manufacturer has the following functions:

- Cost: $C(x) = 10,000 + 50x + 0.5x^2$
- Revenue: $R(x) = 200x - 0.5x^2$

where x is the number of units produced and sold.

a) Find the profit function $P(x) = R(x) - C(x)$.

b) Find the marginal cost, marginal revenue, and marginal profit functions.

c) Determine the production level where marginal revenue equals marginal cost.

d) Verify that this is the same production level where marginal profit equals zero.

e) Calculate the actual profit at this optimal production level.

f) Create a graph showing all three marginal functions on the same axes.

Problem 7: Sensitivity and Error Analysis (xxx)

A pharmaceutical company uses the formula $D(t) = \frac{100t}{t^2+4}$ to model the concentration of a drug in the bloodstream (in mg/L) t hours after administration.

a) Find $D'(t)$ using the quotient rule.

b) Evaluate $D(2)$ and $D'(2)$. Interpret both values.

c) Use linear approximation to estimate $D(2.1)$.

d) The angle of inclination of the tangent line at $t = 2$ is $\alpha = \arctan(D'(2))$. Calculate α in degrees.

e) At what time t is the rate of change of drug concentration equal to zero? What is the concentration at that time?

f) Create a graph showing the concentration function and the tangent line at $t = 2$.