Session 04-05: Tasks

Rational & Logarithmic Functions

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Problem 1: Basic Asymptote Identification (x)

For each rational function, identify all vertical and horizontal asymptotes:

a) $f(x) = \frac{3}{x-2}$

b) $g(x) = \frac{2x+1}{x-3}$

c) $h(x) = \frac{x^2+1}{x^2-4}$

d) $k(x) = \frac{x^3+2x}{x^2+1}$

Problem 2: Holes vs Asymptotes (x)

Identify whether each function has a hole or vertical asymptote at the given point:

a) $f(x) = \frac{x^2-9}{x-3}$ at $x = 3$

b) $g(x) = \frac{x^2-4x+4}{x-2}$ at $x = 2$

c) $h(x) = \frac{x^2-1}{(x-1)(x+2)}$ at $x = 1$

Problem 3: Average Cost Analysis (xx)

A company has total cost function $C(x) = 3600 + 24x + 0.01x^2$ dollars for producing x units.

a) Find the average cost function $AC(x)$
b) Find the production level that minimizes average cost
c) What is the minimum average cost?
d) Find $\lim_{x\to\infty} AC(x)$ and interpret its meaning

Problem 4: Logarithmic Properties (xx)

Simplify each expression using logarithm properties:

a) $\log_3(27x^2)$

b) $\ln(e^{2x} \cdot \sqrt{x})$

c) $2\log_5(5x) - \log_5(x^2)$

d) $\log_2(8) + \log_2(x/4)$

Problem 5: Graph Sketching (xx)

Sketch the rational function $f(x) = \frac{2x-4}{x+1}$ by finding:

a) Domain and asymptotes
b) x and y intercepts
c) Sign analysis
d) End behavior

Problem 6: Logarithmic Equation (xxx)

Solve the equation: $\log_3(x+8) + \log_3(x) = 2$

Problem 7: Semi-log Data Analysis (xxx)

A bacteria culture shows the following population data:

Time (hours)	0	2	4	6	8
Population	100	400	1600	6400	25600

a) Show that this represents exponential growth
b) Find the growth formula $P(t) = P_0 \cdot b^t$
c) What is the doubling time?
d) Predict the population at $t = 10$ hours

Problem 8: Complex Rational Function (xxxx)

Analyze the function $f(x) = \frac{x^2-4}{x-1}$ completely:

a) Find all asymptotes and holes
b) Find the x and y intercepts
c) Find where $f(x) = x + 2$
d) Sketch the function showing all key features