Session 04-03 - Tasks

Exponential Functions Deep Dive

Exponential Functions - Problem Set

Problem 1: Basic Exponential Evaluation (x)

Given the exponential function $f(x) = 3 \cdot 2^x$:

a) Calculate $f(0)$, $f(2)$, and $f(-1)$

b) Find the y-intercept of the function

c) Determine if this represents growth or decay

Problem 2: Identifying Exponential Functions (x)

Determine which of the following are exponential functions. If yes, identify the base and initial value:

a) $g(x) = 5^{2x}$

b) $h(x) = x^5$

c) $k(x) = 4 \cdot (0.7)^x$

d) $m(x) = 2^x + 3$

Problem 3: Bacteria Growth Model (xx)

A bacteria culture starts with 500 cells and doubles every 3 hours.

a) Write an exponential model for the number of bacteria after t hours

b) How many bacteria will there be after 9 hours?

c) How long will it take to reach 32,000 bacteria?

Problem 4: Compound Interest Comparison (xx)

You want to invest €8,000 for 6 years. Compare these options:

- Bank A: 4.5% annual interest, compounded quarterly
- Bank B: 4.4% annual interest, compounded monthly
- Bank C: 4.3% annual interest, compounded continuously

Which option yields the highest return?

Problem 5: Radioactive Decay (xx)

A radioactive isotope has a half-life of 8 days. Starting with 120 grams:

a) Write the exponential decay model

b) How much remains after 24 days?

c) When will only 15 grams remain?

Problem 6: Market Penetration Model (xx)

A new smartphone app launches with 1,000 users. The user base doubles every 2 months.

a) Write the exponential growth model

b) Project the number of users after 6 months

c) How many users will there be after 10 months?

Problem 7: Comparing Investment Strategies (xxx)

You have €10,000 to invest. Compare three investment scenarios:

a) Calculate the value of each investment after 4 years:

- Option A: 6% annual interest, compounded annually
- Option B: 5.8% annual interest, compounded semi-annually
- Option C: 5.7% annual interest, compounded quarterly

b) Option A grows by exactly 50% after how many full years? (Check: when does it first exceed €15,000?)

c) If you need exactly €16,000 after 8 years, which option(s) will achieve this goal?

Problem 8: COVID-19 Spread Analysis (xxxx)

During early 2020, COVID-19 cases in a region grew exponentially before interventions. The data shows: - Day 0: 64 confirmed cases - Cases doubled every 3 days initially - After day 12, strong interventions reduced the growth to 20% every 3 days - After day 24, lockdown reduced growth to 5% every 3 days

a) Write the exponential models for each phase

b) Calculate the number of cases at days 12, 24, and 30

c) How many cases would there have been at day 30 without any interventions?

d) By what factor did the interventions reduce the case count at day 30?