

Tasks 04-02 - Power Functions & Roots

Section 04: Advanced Functions

Problem 1: Power Function Basics (x)

For each function, state the domain and identify whether it's even, odd, or neither:

- a) $f(x) = x^4$
- b) $g(x) = x^{-3}$
- c) $h(x) = x^{2/3}$
- d) $p(x) = \sqrt{x}$
- e) $q(x) = x^{-2}$
- f) $r(x) = \sqrt[3]{x}$

Problem 2: Evaluating Power Functions (x)

Calculate the following values without a calculator:

- a) $f(x) = x^{3/2}$, find $f(4)$ and $f(9)$
- b) $g(x) = x^{-1/2}$, find $g(16)$ and $g(25)$
- c) $h(x) = x^{2/3}$, find $h(8)$ and $h(27)$
- d) $p(x) = 2x^{-2}$, find $p(1/2)$ and $p(-3)$

Problem 3: Economies of Scale Application (xx)

A manufacturing company has a cost function:

$$C(x) = 2000 + 150x^{0.75}$$

where x is the number of units produced (in thousands) and $C(x)$ is the cost in thousands of euros.

- a) Calculate the total cost of producing 8,000 units.
- b) Calculate the average cost per unit when producing 8,000 units.
- c) Compare the average cost per unit for 1,000 units versus 16,000 units. What does this tell you about economies of scale?
- d) If the company sells each unit for €0.50, find the production level where revenue equals €3,000 (in thousands).

Problem 4: Growth Rate Comparison (xx)

Order the following functions from slowest to fastest growth for large positive values of x :

$$f(x) = x^{0.5}, g(x) = x^{1.5}, h(x) = x, p(x) = x^{-0.5}, q(x) = x^2$$

Then evaluate each function at $x = 4$ and $x = 100$ to verify your ordering.

Problem 5: Container Design (xxx)

A company manufactures cubic and cylindrical containers.

Cubic container: Side length s

- Surface area: $S_c = 6s^2$
- Volume: $V_c = s^3$

Cylindrical container: Radius r and height $h = 2r$

- Surface area: $S_{cyl} = 2\pi r^2 + 4\pi r^2 = 6\pi r^2$
- Volume: $V_{cyl} = \pi r^2 \times 2r = 2\pi r^3$

- For a cubic container with volume 64 cubic meters, find the surface area.
- For a cylindrical container with the same volume (64 cubic meters), find the radius and surface area.
- Which shape uses less material (smaller surface area) for the same volume?
- The material costs €10 per square meter. Calculate the material cost difference between the two containers.

Problem 6: Power Function Application (xxxx)

A company's profit function combines multiple power terms:

$$P(x) = -x^2 + 8x^{1.5} - 12x^{0.5} + 100$$

where x is the production level in thousands of units ($x > 0$) and $P(x)$ is profit in thousands of euros.

- Rewrite the function by factoring out $x^{0.5}$ from the first three terms.
- Calculate the profit for production levels of 1, 4, and 9 thousand units.
- Determine which term dominates for small x (near 0) and which dominates for large x .
- Based on your calculations, estimate the production level that might maximize profit.