

Tasks 04-01 - Polynomial Functions

Section 04: Advanced Functions

Problem 1: Polynomial Identification (x)

Determine which of the following are polynomial functions. For those that are, state the degree and leading coefficient:

- a) $f(x) = 3x^4 - 2x^2 + 7x - 1$
- b) $g(x) = \frac{2}{x} + x^3 - 5$
- c) $h(x) = \sqrt{2}x^5 - \pi x^2 + e$
- d) $p(x) = (x - 3)(x + 2)(x - 1)$
- e) $q(x) = x^{2.5} + 3x - 1$
- f) $r(x) = \frac{x^3 - 2x + 1}{2}$

Problem 2: End Behavior Analysis (x)

Without graphing, describe the end behavior of each polynomial:

- a) $P(x) = 4x^6 - 3x^4 + 2x - 7$
- b) $Q(x) = -2x^5 + 8x^3 - x + 10$
- c) $R(x) = -\frac{1}{3}x^8 + 5x^5 - 2x^2$
- d) $S(x) = 7x^3 - 4x^2 + x - 9$

Problem 3: Zeros and Multiplicities (xx)

For each factored polynomial:

- List all zeros and their multiplicities
- Describe whether the graph crosses or touches at each zero
- Find the y-intercept
- State the degree and end behavior

- a) $P(x) = 3(x - 2)^2(x + 1)(x - 4)$
- b) $Q(x) = -2x^3(x + 3)^2$
- c) $R(x) = \frac{1}{2}(x + 2)^3(x - 1)^2(x - 3)$

Problem 4: Manufacturing Cost Analysis (xx)

A factory producing electronic components has a monthly cost function:

$$C(x) = 0.5x^3 - 12x^2 + 90x + 200$$

where x is production in thousands of units and $C(x)$ is cost in thousands of euros.

- a) What are the fixed costs (costs when production is zero)?
- b) Calculate the cost of producing 5,000 units.
- c) Find $C(10) - C(8)$ and interpret this difference.
- d) The company breaks even when revenue equals cost. If revenue is $R(x) = 100x$, show that there's a break-even point at exactly 4,000 units.

Problem 5: Market Share Analysis (xxx)

Three companies compete in a market. Company A's market share over time is modeled by:

$$S(t) = -t^3 + 6t^2$$

where t is years since product launch ($0 \leq t \leq 4$) and $S(t)$ is market share in percentage.

- a) Find $S(0)$, $S(2)$, and $S(4)$. Interpret each value.
- b) Factor the market share function completely.
- c) Find all times when Company A has zero market share.
- d) Based on the factored form and a sign analysis, determine when Company A has positive market share.

Problem 6: Polynomial Sketching (xx)

Sketch the graph of $P(x) = -\frac{1}{4}(x+3)(x-1)^2(x-4)$ by following these steps:

- a) Identify all zeros and their multiplicities
- b) Determine the end behavior
- c) Find the y-intercept
- d) Determine the sign of $P(x)$ in each interval between zeros
- e) Sketch the complete graph

Problem 7: Business Application - Profit Analysis (xxx)

A company's quarterly profit (in thousands of euros) is modeled by:

$$P(x) = (x-1)(x-3)(x-5)$$

where x represents the quarter number ($1 \leq x \leq 8$).

- a) Expand the polynomial into standard form.
- b) Find all break-even quarters (when profit = 0).
- c) Calculate the profit for quarters 2, 4, and 6.
- d) Determine during which quarters the company experiences losses.
- e) Without using calculus, estimate which quarter likely has the highest profit.

Problem 8: Intermediate Value Theorem (xxxx)

Consider the polynomial $P(x) = x^4 - 5x^2 + 2x + 3$.

- a) Calculate $P(-2)$, $P(-1)$, $P(0)$, $P(1)$, and $P(2)$.
- b) Use the Intermediate Value Theorem to identify at least two intervals that must contain zeros.
- c) Show that $P(x)$ can be rewritten as $(x^2 - 3)(x^2 - 2) + 2x - 3$ by expanding this expression.
- d) Based on the degree and complexity of $P(x)$, explain why finding its exact zeros requires advanced techniques beyond what we've studied.