Tasks 03-05 - Composition, Inverses & Advanced Graphing

Section 03: Functions as Business Models

Problem 1: Function Composition Basics (x)

Given the functions: - $f(x) = 3x - 2$ - $g(x) = x^2 + 1$ - $h(x) = \sqrt{x}$

Find: a) $(f \circ g)(x)$ b) $(g \circ f)(x)$ c) $(f \circ g)(3)$ d) $(g \circ f)(2)$ e) $(h \circ g)(4)$

Problem 2: Finding Inverse Functions (x)

Find the inverse function for each of the following, and state the domain and range of both the original and inverse:

a) $f(x) = 4x - 7$

b) $g(x) = \frac{x+2}{3}$

c) $h(x) = 2x^3 - 1$

d) $p(x) = \frac{5}{x-3}$ where $x \neq 3$

Problem 3: Production Process Composition (xx)

A manufacturing company has a three-stage production process:

Stage 1 (Raw Material Processing): - Cost: $C_1(x) = 30x + 150$ where x is kg of raw material

Stage 2 (Component Manufacturing): - Produces: 0.7 kg of components per kg processed - Additional cost: $C_2(y) = 50y + 200$ where y is kg of components

Stage 3 (Final Assembly): - Each kg of components yields 5 products - Packaging cost: $C_3(z) = 2z + 100$ where z is number of products

a) Express the total cost as a function of raw material quantity x.

b) How much raw material is needed to produce 100 final products?

c) What is the total cost for producing 100 final products?

d) Find the average cost per product when starting with 10 kg of raw material.

Problem 4: Currency Exchange and Investment (xx)

An investor uses the following process: - Converts USD to EUR: $E(x) = 0.85x$ - Invests EUR with return function: $R(y) = 1.08y - 50$ (minimum €50 fee) - Converts back to USD: $U(z) = 1.18z$

a) Express the final USD amount as a function of initial USD investment x.

b) What is the minimum USD investment needed to avoid losing money?

c) Find the inverse of the overall process. What does it represent?

d) If an investor wants to end with exactly $10,000 USD, how much should they initially invest?

Problem 5: Supply Chain Optimization (xxx)

A retailer has the following functions in their supply chain:

Wholesale Purchase: - Price per unit depends on quantity: $P(x) = 100 - 0.5x$ (for $x \leq 150$) - Total wholesale cost: $W(x) = x \cdot P(x)$

Processing & Storage: - Processing reduces quantity by 5% (damage/loss) - Storage cost: $S(y) = 20y + 500$ where y is stored quantity

Retail Sales: - Can sell at: $R(z) = 180z - z^2$ revenue for z units

a) Express the profit as a function of wholesale purchase quantity x.

b) Find the domain restrictions for this business model.

c) If the retailer can only store 100 units, what wholesale quantity maximizes profit?

d) Create an inverse function that determines wholesale quantity needed for a target revenue.

Problem 6: Market Equilibrium with Taxes (xxx)

Consider a market with: - Demand: $Q_d = 1000 - 10p$ - Supply: $Q_s = 5p - 50$

The government implements a tax system: - Tax on consumers: $T_c(p) = p + 5$ (perceived price) - Tax on producers: $T_p(p) = 0.9p$ (received price)

a) Find the original equilibrium without taxes.

b) With consumer tax only, find the new equilibrium and tax revenue.

c) With producer tax only, find the new equilibrium and tax revenue.

d) With both taxes, express the market-clearing condition and solve for equilibrium.

e) Find the inverse demand function and interpret its meaning with taxes.

Problem 7: Comprehensive Business Model (xxxx)

TechStart company has developed a complex business model:

Production: - Fixed costs: €10,000 - Variable cost function: $V(x) = 20x + 0.01x^2$ - Total cost: $C(x) = 10000 + V(x)$

Pricing Strategy: - Base price function: $P(x) = 200 - 0.5x$ - Seasonal adjustment: $S(p, t) = p(1 + 0.2\sin(\frac{\pi t}{6}))$ where t is month

Demand: - At price p: $D(p) = 2000 - 8p$ - Quality factor: Higher quality increases demand by factor $Q(x) = 1 + 0.001x$

Multi-Channel Sales: - Online: 60% of sales at full price - Retail: 40% of sales at 90% of price

a) Express profit as a function of production quantity x (ignore seasonal effects).

b) Find the production level where average cost equals marginal revenue.

c) If the company wants demand of exactly 1000 units, what price should they set?

d) During month 3 (peak season), how does the optimal price change?

e) Create a composite function showing how production quantity affects final demand through the price mechanism.

f) If the company can only produce between 50 and 150 units due to capacity constraints, find the optimal production level and explain whether these constraints are binding.

Submission Instructions

- Complete all problems showing clear working
- Pay special attention to composition order and domain restrictions
- For inverse functions, always verify your answer
- Submit at the beginning of Section 04
- Be prepared to present any solution to the class

Estimated completion time: 3-4 hours

Congratulations on completing Section 03! These problems integrate all concepts from functions as business models. Your mastery of these tools will be essential for the advanced functions in Section 04!