Tasks 03-04 - Transformations & Graphical Analysis

Section 03: Functions as Business Models

Problem 1: Basic Transformations (x)

Given the function $f(x) = x^2 - 2x + 3$, write the equation for each transformation:

a) $f(x)$ shifted up by 4 units

b) $f(x)$ shifted left by 3 units

c) $f(x)$ reflected over the x-axis

d) $f(x)$ stretched vertically by a factor of 2

e) $f(x)$ shifted right by 2 units and down by 5 units

Problem 2: Cost Function Adjustments (xx)

A manufacturing company has a cost function $C(x) = 0.02x^2 + 15x + 2000$ where x is the number of units produced.

> Tip
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> - The $0.02x^2$ term represents increasing inefficiency at high volumes
> - The $15x$ term represents variable costs per unit (materials, labor per unit)
> - The 2000 represents fixed costs (rent, utilities, base salaries)

a) Due to a new supplier contract, variable costs (both the $0.02x^2$ and $15x$ terms) decrease by 20%. Write the new cost function.

b) The company moves to a larger facility, increasing fixed costs by €1,500. Write the adjusted cost function starting from the original.

c) Combining both changes from parts (a) and (b), what is the final cost function?

d) Calculate the cost for producing 100 units using both the original and final functions. Which is cheaper?

e) At what production level do the original and final cost functions have the same total cost? What does this mean for the company's decision?

Problem 3: Demand Curve Transformations (xx)

A product's weekly demand is modeled by $D(p) = 1000 - 20p$ where p is the price in euros.

a) During a promotional campaign, demand increases by 30% at every price level. Write the new demand function.

b) After the promotion ends, a competitor's product launch causes the demand curve to shift, reducing demand by 200 units at every price. Write this demand function.

c) Economic conditions change consumer behavior: due to inflation, when the actual price is p euros, consumers now perceive it as being 10% more expensive and adjust their purchasing accordingly. How does this transform the original demand function? Hint: If consumers perceive the price as 10% higher, they react as if the price were $1.1p$ euros.

d) Find the price that generates demand of 500 units for:

- The original function
- The promotional function
- The post-competitor function

Problem 4: Seasonal Business Model (xxx)

An ice cream shop's monthly profit follows the function:

$$P(m) = -200(m-7)^2 + 8000$$

where m is the month (1 = January, 12 = December).

a) In which month does the shop achieve maximum profit? What is this profit?

b) Due to climate change, the peak season shifts 1.5 months earlier and maximum profit increases by 15%, everything else stays the same. Write the new profit function.

c) The shop opens a second location in the Southern Hemisphere. In the Southern Hemisphere, summer occurs during Northern winter. Hence, when it's July (peak summer) in the North, it's January (peak winter) in the South. Write the profit function for the Southern location, assuming it has the same profit pattern from task a. but peaks in January instead of July.

d) If both locations (North and South) operate simultaneously, write the combined monthly profit function $P_{total}(m)$ that represents total profit across both shops based on task a.

e) Using the combined function from part (d), find which months the total profit exceeds €10,000.

Problem 5: Graph Interpretation (xxx)

You are given a graph showing three functions:

- Function A: Linear, passing through (0, 100) and (50, 600)
- Function B: Quadratic, vertex at (30, 700), passing through origin
- Function C: Linear, passing through (0, 400) and (40, 400)

These represent cost functions for three different production methods.

a) Write the equation for each function.

b) Determine the production ranges where each method is most cost-effective.

c) A company expects to produce 35 units. Which method should they choose and what will be the cost?

d) If fixed costs for Method A increase by €200, how does this change your recommendation for 35 units?

Problem 6: Revenue Transformation Analysis (xxx)

A software company's monthly revenue from selling x subscriptions is:

$$R(x) = -2x^2 + 120x$$

where x is the number of subscriptions sold (in hundreds) and revenue is in euros.

> Understanding the function
>
> - Revenue depends on quantity sold
> - The quadratic form reflects diminishing returns (marketing costs, market saturation)
> - Each subscription currently sells for the same price

The company is analyzing different market scenarios and how they affect revenue.

a) Find the optimal number of subscriptions to sell that maximizes revenue. What is the maximum monthly revenue?

b) The company improves their product features. Market research shows this will increase revenue per subscription by 15% across all quantity levels (customers perceive higher value). Write the transformed revenue function $R_{improved}(x)$ and find the new optimal quantity and maximum revenue.

c) Due to increased competition, the company expects overall revenue to drop by 25% at every quantity level. Write the transformed revenue function $R_{competition}(x)$ and find the new maximum revenue. How much revenue is lost compared to the original?

d) The company considers an aggressive marketing campaign. This is expected to boost revenue by 30% at all quantity levels (better brand awareness, higher perceived value). Write the transformed revenue function $R_{marketing}(x)$ and find its maximum revenue.

e) Based on your calculations, rank the scenarios from best to worst in terms of maximum revenue. What do you notice about the optimal quantity in each scenario?

Problem 7: Multi-Location Profit Analysis (xxxx)

A restaurant chain has a successful location with profit function:

$$P(d) = -5d^2 + 200d - 1500$$

where d is the number of daily customers (in tens).

They plan to open three new locations with different market conditions:

Location A (Downtown):

- Market research suggests profits will be 20% higher at every customer level (premium neighborhood)
- Fixed costs increase by €500 (expensive rent)

Location B (Suburbs):

- Profits expected to be 30% lower at every customer level (lower prices, higher costs)
- Fixed costs reduced by €200 (cheaper rent)

Location C (Mall):

- Peak customer time occurs 5 units earlier than the original (lunch rush vs dinner rush)
- Profits are 10% lower at every customer level
- Fixed costs increase by €300

a) Write the transformed profit function for each location (A, B, and C).

b) Find the optimal number of customers and maximum profit for each location.

c) If the chain can only open two new locations, which combination maximizes total daily profit?

d) For Location C specifically, explain how the horizontal shift affects when the restaurant reaches peak profit during the day.