Tasks 03-03 - Quadratic Functions & Basic Optimization

Section 03: Functions as Business Models

Problem 1: Vertex and Properties (x)

For each quadratic function, find the vertex, axis of symmetry, and determine whether it has a maximum or minimum value:

a) $f(x) = x^2 - 6x + 5$

b) $g(x) = -2x^2 + 8x - 3$

c) $h(x) = 3x^2 + 12x + 15$

d) $p(x) = -\frac{1}{2}x^2 + 4x - 6$

Problem 2: Completing the Square (x)

Convert each quadratic function from standard form to vertex form by completing the square:

a) $f(x) = x^2 + 10x + 21$

b) $g(x) = 2x^2 - 8x + 5$

c) $h(x) = -x^2 + 6x - 7$

d) $k(x) = 3x^2 - 18x + 24$

Problem 3: Revenue Optimization (xx)

A theater company is planning ticket prices for a new show. Market research indicates:

- At €20 per ticket, they expect 500 attendees
- For each €2 increase in price, attendance drops by 20 people
- The theater has a capacity of 600 seats

a) Express the attendance A as a function of ticket price p.

b) Write the revenue function $R(p)$.

c) Find the ticket price that maximizes revenue.

d) What is the maximum revenue and how many tickets will be sold?

e) Verify that the optimal solution respects the theater capacity.

Problem 4: Profit Maximization (xx)

A smartphone accessories company manufactures premium cases. Their market analysis shows:

- Demand: $Q = 2000 - 25p$ (units per month)
- Fixed costs: €15,000 per month
- Variable cost: €20 per case

a) Write the revenue function $R(p)$ in terms of price.

b) Write the cost function $C(p)$ in terms of price.

c) Write the profit function $\Pi(p)$ and identify its form.

d) Find the profit-maximizing price and quantity.

e) Calculate the maximum monthly profit.

f) Determine the break-even prices.

Problem 5: Area Optimization (xxx)

A farmer has 400 meters of fencing to create a rectangular grazing area along a straight river. The side along the river needs no fence.

a) Let x be the length perpendicular to the river. Express the area A as a function of x.

b) Find the dimensions that maximize the grazing area.

c) What is the maximum area?

d) If the farmer needs at least 8,000 m² for the animals, what range of widths x will work?

e) Due to terrain, the length parallel to the river cannot exceed 150 meters. How does this affect the optimal dimensions?

Problem 6: Projectile Motion Application (xxx)

A company is testing delivery drones that follow parabolic flight paths. One drone's height h (in meters) after t seconds is given by:

$$h(t) = -2t^2 + 16t + 10$$

a) Find the maximum height reached by the drone.

b) At what time does it reach maximum height?

c) When does the drone return to its starting height of 10 meters?

d) If the drone must maintain a minimum height of 30 meters for safety, during what time interval is it safe?

e) The drone's horizontal speed is 5 m/s. How far does it travel horizontally while above 30 meters?

Problem 7: Multi-Product Optimization (xxxx)

TechGear sells two related products: wireless earbuds and charging cases. Market research reveals:

Earbuds:

- Demand: $Q_e = 1000 - 5p_e$ when sold alone
- Production cost: €30 per unit
- Can be sold with or without case

Charging Cases:

- Only bought by earbud customers
- 60% of earbud buyers will buy a case if priced at €20
- For each €5 increase in case price, 10% fewer earbud buyers purchase a case
- Production cost: €8 per case

Bundle Option:

- The company is considering bundling both products
- Bundle demand: $Q_b = 800 - 4p_b$ where p_b is bundle price
- Same production costs apply

a) Write the function for case demand Q_c as a percentage of earbud sales, depending on case price p_c.

b) If earbuds are priced at €80 and cases at €25, calculate:

   - Number of earbuds sold
   - Number of cases sold
   - Total profit from this pricing strategy

c) Find the optimal bundle price and calculate the profit from bundling.

d) The company can only produce 500 units total (earbuds + cases) per month. Which strategy should they pursue:

   - Separate pricing at €80 for earbuds and €25 for cases
   - Bundling at the optimal bundle price
   - Another pricing strategy you determine
   - Justify your answer with calculations.