# Tasks 03-02 - Linear Functions & Economic Applications

## Section 03: Functions as Business Models

## Problem 1: Linear Function Forms (x)

Convert the following linear functions between different forms:

a) Given the points (3, 17) and (7, 29), find the equation in slope-intercept form.

b) Rewrite $3x - 4y = 24$ in slope-intercept form.

c) A line passes through (5, 20) with slope $m = -3$. Write the equation in:

- Point-slope form
- Slope-intercept form

d) Find the equation of a line parallel to $y = 2x - 7$ that passes through the point (4, 10).

## Problem 2: Supply and Demand Analysis (xx)

A local electronics store sells wireless headphones. Market research provides:

- Demand function: $Q_d = 500 - 4p$ (units per month)
- Supply function: $Q_s = 100 + 6p$ (units per month)

where $p$ is the price in euros and $Q$ is the quantity.

a) Find the market equilibrium price and quantity.

b) At a price of €30, is there a shortage or surplus? Calculate the amount.

c) What is the maximum price consumers would theoretically pay (when demand = 0)?

d) Below what price will suppliers not provide any headphones?

e) If a €10 tax per unit is imposed on suppliers, the new supply becomes $Q_s = 100 + 6(p - 10)$. Find the new equilibrium.

## Problem 3: Cost-Volume-Profit Analysis (xx)

A startup producing eco-friendly water bottles has the following cost structure:

- Fixed monthly costs: €12,000 (rent, equipment lease, insurance)
- Variable cost per bottle: €8 (materials and labor)
- Selling price per bottle: €20

a) Write the cost function $C(x)$ and revenue function $R(x)$ where $x$ is the number of bottles.

b) Calculate the contribution margin per bottle and explain its meaning.

c) Find the break-even point in units and in euros.

d) How many bottles must be sold to achieve a profit of €6,000?

e) If fixed costs increase by €3,000, how does this affect the break-even point?

## Problem 4: Linear Depreciation (xx)

A delivery company purchases vehicles and equipment with different depreciation schedules:

- Van: Initial value €40,000, depreciates €5,000 per year
- Motorcycles: Initial value €8,000 each, depreciate €2,000 per year
- Computer system: Initial value €15,000, depreciates €3,000 per year

a) Write the value function $V(t)$ for each asset type.

b) After how many years will each asset be fully depreciated?

c) What is the total value of 1 van, 3 motorcycles, and the computer system after 3 years?

d) The company wants to sell the van when its value drops to 40% of the original price. When should they sell?

## Problem 5: Market Competition (xxx)

Two competing internet service providers operate in the same city:

TechNet:

- Fixed monthly fee: €15
- Cost per GB of data: €0.50

SpeedLink:

- Fixed monthly fee: €25
- Cost per GB of data: €0.30

a) Write the total monthly cost function for each provider as a function of data usage $x$ (in GB).

b) For what data usage do both providers cost the same?

c) Create a recommendation guide: Which provider is better for different usage levels?

d) If TechNet reduces its per-GB cost to €0.40, how does this change the break-even point?

e) SpeedLink introduces a premium plan: €40 fixed fee with €0.20 per GB. For heavy users (>100 GB/month), which of the three options is best?

## Problem 6: Production Planning (xxx)

A furniture workshop produces tables and chairs with the following constraints and information:

Production requirements:

- Each table requires 4 hours of carpentry and 2 hours of finishing
- Each chair requires 2 hours of carpentry and 1 hour of finishing
- Available per week: 160 hours carpentry, 70 hours finishing

Financial data:

- Table: sells for €300, costs €180 to make
- Chair: sells for €150, costs €90 to make

a) Write the constraint inequalities for production planning.

b) If the workshop produces 30 tables, what is the maximum number of chairs possible?

c) Express profit as a function of the number of tables ($t$) and chairs ($c$) produced.

d) Find the profit if the workshop operates at full capacity making only chairs.

e) The workshop receives an order for exactly 25 tables and 40 chairs. Can they fulfill this order in one week? If not, what percentage of capacity would be required?

## Problem 7: Dynamic Pricing Strategy (xxxx)

An online streaming service is analyzing its pricing model. Market research shows:

- At €5/month: 100,000 subscribers
- At €10/month: 75,000 subscribers
- At €15/month: 50,000 subscribers

The company has:

- Fixed monthly costs: €200,000 (servers, licenses, staff)
- Variable cost per subscriber: €2/month (bandwidth, support)

a) Assuming a linear demand relationship, find the demand function $S(p)$ where $S$ is subscribers and $p$ is price.

b) Express the company's monthly revenue $R(p)$ as a function of price.

c) Express the company's monthly profit $\Pi(p)$ as a function of price.

d) Using trial values, estimate the price that maximizes profit (we'll learn the exact method in session 03-03).

e) The company considers a two-tier model:

   - Basic: €8/month with expected 80,000 subscribers
   - Premium: €15/month with expected 30,000 subscribers
   - Additional fixed costs for maintaining two tiers: €50,000/month

   Compare the profit of this two-tier model to a single price of €10/month.