Tasks 03-01 - Function Concepts & Business Modeling

Section 03: Functions as Business Models

Problem 1: Creating Tables and Graphs (xx)

A streaming service charges according to the function $S(h) = 10 + 2h$ where h is the number of hours watched per month and $S(h)$ is the total cost in euros.

a) Create a table showing the cost for $h = 0, 5, 10, 15, 20, 25$ hours
b) Sketch a graph of this function on graph paper or using graphing software
c) What does 10 represent?
d) If a customer has a budget of €30 per month, how many hours can they watch?

Problem 2: Domain and Range (xx)

Find the domain and range of each function:

a) $f(x) = \frac{2x+3}{x-4}$
b) $g(x) = \sqrt{x+5}$
c) $h(x) = 3x - 8$
d) $p(x) = \frac{1}{x^2+1}$

Problem 3: Bakery Business Functions (xx)

A local bakery has the following cost structure: - Monthly rent and utilities: €2,500 - Ingredients and materials per cake: €12 - Labor cost per cake: €8 - Each cake sells for €45

a) Define the cost function $C(x)$ where x is the number of cakes produced per month
b) Define the revenue function $R(x)$
c) Define the profit function $P(x)$
d) Create a table showing $C(x)$, $R(x)$, and $P(x)$ for $x = 0, 50, 100, 150, 200$ cakes
e) Sketch graphs of all three functions on the same axes
f) How many cakes must be sold to break even? (Show this on your graph)
g) If the bakery can produce a maximum of 200 cakes per month, what is the maximum possible profit?

Problem 4: Manufacturing Constraints (xxx)

A small electronics manufacturer produces two types of devices: tablets and smartphones. The production process has the following constraints:

- Each tablet requires 3 hours of assembly time and 2 hours of testing
- Each smartphone requires 2 hours of assembly time and 1 hour of testing

- The factory has 120 hours of assembly time available per week
- The factory has 60 hours of testing time available per week
- Tablets sell for €300 with a production cost of €180
- Smartphones sell for €200 with a production cost of €110

a) Let t represent tablets produced and s represent smartphones produced. Write the constraint inequalities.
b) Express the total revenue R as a function of t and s
c) Express the total profit P as a function of t and s
d) If the company decides to produce only tablets, what is the maximum number they can produce per week? What would be the profit?
e) Can the company produce 30 tablets and 20 smartphones in one week? Justify your answer.

Problem 5: Investment Portfolio Analysis (xxxx)

An investment advisor is creating a model for a client's portfolio. The client can invest in three options:

- Bonds: Fixed return of 4% per year
- Stocks: Variable return modeled by $r_s(x) = 0.12 - 0.0001x$ where x is the amount invested in thousands of euros
- Real Estate Fund: Return rate of $r_e(x) = \frac{8}{100+0.01x}$ where x is the amount in thousands of euros

The client has €50,000 to invest total.

a) If the client invests €20,000 in bonds, €20,000 in stocks, and €10,000 in real estate, calculate the expected annual return in euros.

b) Find the domain for the stock return function $r_s(x)$ that ensures a positive return rate.

c) Explain why the real estate return function $r_e(x)$ shows diminishing returns as investment increases.

d) If the client wants to split the investment equally between bonds and stocks only (€25,000 each), compare this to investing all €50,000 in bonds. Which strategy yields higher returns?