Tasks 02-05 - Exponential, Logarithmic & Complex Word Problems

Section 02: Equations & Problem-Solving Strategies

Instructions

Complete these problems to master advanced exponential and logarithmic equations, as well as complex multi-step word problems. These problems integrate all equation-solving techniques from Section 02.

Problem 1: Exponential Equations (x)

Solve each exponential equation:

a) $2^{x+3} = 128$

b) $5^x \cdot 25^{x-1} = 125$

c) $3^{2x} - 12 \cdot 3^x + 27 = 0$

d) $4^x - 2^{x+1} - 8 = 0$

Problem 2: Logarithmic Equations (x)

Solve each logarithmic equation, stating domain restrictions:

a) $\log_3(x + 4) = 2$

b) $\log(x) + \log(x + 3) = 1$

c) $\log_2(x) - \log_8(x) = 1$

d) $\log_x(16) = 4$

Problem 3: Mixed Exponential-Logarithmic (xx)

Solve these equations that involve both exponential and logarithmic expressions:

a) $3^{\log_3(x)} + x = 10$

b) $\log_2(2^x + 1) = x - 1$

> 💡 Tip
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> Note, there might be multiple solutions or no solution at all!

Problem 4: Growth and Decay Applications (xx)

A radioactive substance decays according to $A(t) = A_0 e^{-kt}$ where t is in years.

a) If 70% remains after 5 years, find the decay constant k
b) Find the half-life of the substance
c) How long until only 10% remains?
d) If we start with 100 grams, when will exactly 25 grams remain?

Problem 5: Investment Comparison (xx)

An investor has €30,000 to invest. She's considering three options:

- Option A: 6% annual interest, compounded yearly
- Option B: 5.8% annual interest, compounded monthly
- Option C: 5.7% annual interest, compounded continuously

a) Write the formula for each option after t years
b) Which option yields the most after 10 years?
c) If she wants €50,000, how long would each option take?

Problem 6: Population Dynamics (xxx)

Two bacterial cultures are growing in a lab. Culture A starts with 500 bacteria and doubles every 3 hours. Culture B starts with 800 bacteria and grows according to $B(t) = 800e^{0.15t}$ where t is in hours.

a) Write the growth equation for Culture A
b) When will the populations be equal?