Tasks 02-02 - Systems of Linear Equations & Matrices

Section 02: Equations & Problem-Solving Strategies

Instructions

Complete these problems to master solving systems of linear equations and basic matrix operations. Practice choosing the appropriate solution method (substitution, elimination, or matrix methods) based on the system's structure.

Problem 1: Method Selection Practice (x)

Solve each 2×2 system using the indicated method:

a) Use substitution:

$$x - 2y = 8$$
$$3x + y = 5$$

b) Use elimination:

$$2x + 5y = 16$$
$$3x + 5y = 21$$

c) Your choice of method:

$$4x + 3y = 25$$
$$2x - y = 5$$

Problem 2: Special Cases Recognition (x)

Without fully solving, identify whether each system has a unique solution, no solution, or infinite solutions. Then verify your answer by solving.

a) $3x - 6y = 12$
$x - 2y = 4$

b) $2x + 4y = 10$
$3x + 6y = 18$

c) $x + 3y = 7$
$2x - y = 0$

Problem 3: Supply and Demand Equilibrium (xx)

A market for organic coffee has the following supply and demand equations (Q in thousands of pounds, P in dollars):

Market A (Urban):

- Demand: Q = 80 - 4P
- Supply: Q = 2P - 10

Market B (Suburban):

- Demand: Q = 60 - 2P
- Supply: Q = 3P - 15

a) Find the equilibrium price and quantity for each market.

b) If a $3 tax per pound is imposed on suppliers in Market A, find the new equilibrium.

c) At what price would both markets have the same quantity demanded?

Problem 4: Market Share Analysis (xxx)

Three competing coffee chains (A, B, C) have interdependent market shares in a city. Market research shows:

1. Chain A's share plus half of B's share equals 45% of the market
2. Chain B's share is 10 percentage points more than C's share
3. Total market share is 100%

a) Set up the system of equations
b) Determine if the system is consistent and each chain's market share
c) If Chain B exits the market and its customers split equally between A and C, what are the new shares?

Problem 5: Manufacturing Operations

A manufacturer operates two plants to fulfill a contract for 1,000 units:

Plant A:

- Production cost: $12/unit
- Maximum capacity: 500 units
- Shipping cost to customer: $2/unit

Plant B:

- Production cost: $8/unit
- Maximum capacity: 800 units
- Shipping cost to customer: $3/unit

The customer pays $25/unit. How should production be split between plants to maximize profit?

Problem 6: Cost Allocation with Constraints (xxx)

MegaCorp needs to allocate $180,000 in IT infrastructure costs among four departments. The allocation must satisfy these constraints:

1. Sales and Marketing together must pay exactly 45% of the total

2. Operations must pay twice as much as Finance
3. Finance must pay $5,000 more than Marketing
4. Sales must pay 1.5 times what Marketing pays

Determine the exact allocation for each department.

> Tip
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> There might not be feasible solution to this problem!