Tasks 01-05 - Logarithms & Substitution

Advanced Algebraic Tools

Problem 1: Substitution for Factorization

Use substitution to factor the following expressions completely:

a) $x^4 + 5x^2 + 6$

b) $4x^4 - 13x^2 + 9$

c) $x^6 - 9x^3 + 8$

d) $(x^2 + 3x)^2 - 2(x^2 + 3x) - 24$

e) $16x^4 - 8x^2 + 1$

Problem 2: Logarithm Fundamentals

Evaluate the following without a calculator:

a) $\log_2(64)$

b) $\log_3\left(\frac{1}{27}\right)$

c) $\log_5(125)$

d) $\log_{10}(0.001)$

e) $\log_4(8)$ (Hint: Express 8 as a power of 2, and 4 as a power of 2)

f) $\ln(e^3)$

Problem 3: Logarithm Laws

Simplify the following expressions using logarithm laws:

a) $\log_2(16) + \log_2(8) - \log_2(4)$

b) $\log(50) + \log(20) - \log(100)$

c) $3\log_5(x) - \log_5(x^2) + \log_5(25)$

d) $\log_3(81) - 2\log_3(9) + \log_3(27)$

e) Express as a single logarithm: $\frac{1}{2}\ln(x) + 3\ln(y) - \ln(z)$

Problem 4: Solving Logarithmic Equations

Solve the following equations:

a) $\log_3(x + 5) = 2$

b) $\log(2x - 1) - \log(x + 2) = 0$

c) $2^{\log_2(x)} = 8$

d) $\log_x(49) = 2$

Problem 5: Binomial

a) Expand completely: $(2x - 3)^4$

b) Expand completely: $(3x + 1)^6$

Problem 6: Logarithmic Application

a) An investment grows from €5,000 to €8,000 in 6 years with continuous compounding. Find the interest rate.