Session 09-01 - Advanced Functions Review

Section 09: Exam Preparation

Dr. Nikolai Heinrichs & Dr. Tobias Vlćek

Warm-Up Quiz - 10 Minutes

Quick Review Questions

Work individually for 5 minutes, then discuss with the class (5 minutes)

1. Simplify $\left(\ln \frac{e^{5x}}{e^{2x-1}}\right)$.

2. Determine all asymptotes of $f(x) = \frac{3x-6}{x^2-4}$.

3. Describe every transformation that maps e^x to $h(x) = -2e^{x+3} - 5$.

4. Solve $5^{2x+1} = 125^{\,x-1}$ for x.

Guided Practice - Part A - 25 Minutes

Practice Problem 1

Log & Exponent Rules Refresh (Sections 01 & 04)

Work in pairs

(a) Simplify $\left(\ln e^{3x} \cdot \sqrt{x}\,\right)$ completely.

(b) Solve $3^{2x-1} = 27^{\,x}$ for x.

Practice Problem 2

Rational Function End-to-End (Sections 04 & 01)

Work in pairs

Analyze $f(x) = \frac{2x^2-8}{x^2-x-6}$ completely:

(a) Factor numerator and denominator.

(b) State the domain.

(c) Identify any holes and vertical asymptotes.

(d) Find the horizontal asymptote.

(e) Find all zeros of f.

(f) Sketch the graph, marking all key features.

Practice Problem 3

Exponential Function with Transformations (Sections 04 & 03)

Work individually, then discuss

Consider $g(x) = 3 \cdot e^{x-2} + 1$.

(a) Identify all transformations applied to the base function e^x.

(b) State the horizontal asymptote of g.

(c) Find $g(0)$ and $g(2)$.

(d) Find the inverse function $g^{-1}(x)$.

Practice Problem 4

Business Word Problem (Sections 04 & 01)

Work in pairs

A tech startup's market penetration is modeled by

$$M(t) = 40 - 30 \cdot e^{-0.2t} \quad (\text{in } \%)$$

where t is the number of years since launch.

(a) What is the long-term market share?

(b) When does market penetration reach 35%?

(c) At what rate is the market share growing at $t = 5$?

Practice Problem 5

Polynomial Factorization (Sections 04 & 01)

Work individually, then discuss

Consider $p(x) = x^4 - 5x^2 + 4$.

(a) Factor $p(x)$ completely using the substitution $u = x^2$.

(b) Find all zeros of p.

(c) Determine the end behavior of p.

Guided Practice - Part B - 30 Minutes

Practice Problem 6

Inverse Functions with Composition Verification (Sections 03 & 04)

Work in pairs

Consider $f(x) = \frac{2x+3}{x-1}$.

(a) Find the inverse function $f^{-1}(x)$.

(b) Verify that $f(f^{-1}(x)) = x$.

(c) State the domain and range of both f and f^{-1}.

Practice Problem 7

Exponential Decay Word Problem (Sections 04, 01 & 08)

Work in pairs

A machine purchased by a manufacturing company depreciates according to the following where t is measured in years.

$$V(t) = 25000 \cdot e^{-0.15t} \quad \text{(in €)}$$

(a) What is the initial value of the machine?

(b) What is the value of the machine after 5 years?

(c) When does the value drop below €5,000?

(d) What is the half-life of the machine's value?

Practice Problem 8

Piecewise-Defined Rational Function (Section 04)

Work individually, then discuss

Consider $f(x) = \frac{|x^2-4|}{x-2}$.

(a) Simplify $f(x)$ separately for $x > 2$ and $x < 2$.

(b) Sketch the graph of f.

(c) Discuss the continuity of f at $x = 2$.

Coffee Break - 15 Minutes

Collaborative Problem-Solving - Part A - 25 Minutes

Practice Problem 9

Drug Concentration Analysis (Sections 04 & 01)

Think individually (2 min), then work in groups of 3-4

The concentration of a medication in the bloodstream is modeled by

$$C(t) = \frac{200t}{t^2 + 25} \quad \text{(mg/L)}$$

where t is the number of hours after administration.

(a) Find the domain and any asymptotes.

(b) At what time is the concentration maximized?

(c) What happens to the concentration as $t \to \infty$?

(d) After how many hours does the concentration drop below 5 mg/L?

Practice Problem 10

Logarithmic Equation Challenge (Sections 04 & 01)

Think individually (2 min), then work in groups of 3-4

Solve $\log_2(x+3) + \log_2(x-1) = 3$.

Be sure to check domain restrictions and verify your solution.

Practice Problem 11

Competing Products (Sections 04 & 01)

Think individually (2 min), then work in groups of 3-4

Product A's market share follows $S_A(t) = 60 - e^{-0.3t})$ and Product B follows $S_B(t) = 80 - e^{-0.1t})$, where t is measured in years.

(a) Which product reaches 50% market share first?

(b) Find the time when both products have equal market share.

(c) What are the long-term equilibrium shares?

Coffee Break - 10 Minutes

Collaborative Problem-Solving - Part B - 25 Minutes

Practice Problem 12

Population Model with Logistic Growth (Sections 04 & 01)

Think individually (2 min), then work in groups of 3-4

A population grows as follows where t is measured in years.

$$P(t) = \frac{10000}{1 + 49e^{-0.3t}}$$

(a) Show that this is logistic growth by identifying its general form.

(b) Find $P(0)$.

(c) Find the carrying capacity (i.e., $\lim_{t \to \infty} P(t)$).

(d) When does P reach half the carrying capacity?

(e) Find $P'(t)$ and determine the time of maximum growth rate.

Practice Problem 13

System of Exponential Equations (Sections 01 & 02)

Think individually (2 min), then work in groups of 3-4

Solve the system of equations:

$$2^x \cdot 3^y = 72 \qquad \text{and} \qquad \frac{2^x}{3^y} = 2$$

Practice Problem 14

Full Function Comparison (Sections 04 & 03)

Think individually (2 min), then work in groups of 3-4

Compare the two functions $f(x) = \ln(x^2)$ and $g(x) = 2\ln(x)$.

(a) Are they the same function? Explain carefully.

(b) State the domain of each function.

(c) Simplify $f(x)$ and discuss when the simplification $\ln(x^2) = 2\ln(x)$ is valid.

(d) Sketch both functions on the same set of axes.

Wrap-Up & Next Steps

Key Takeaways

- Logarithm rules (product, quotient, power) are essential tools for simplification and equation solving
- Rational functions require careful analysis: factor first, then distinguish holes from asymptotes
- Exponential models appear in depreciation, population growth, and market share — solving them requires logarithms
- Absolute values in function definitions require careful case analysis (e.g., $\ln(x^2) \neq 2\ln(x)$ in general)

. . .

> 💡 Tip
>
> Session 09-02 will cover review of Financial Mathematics (Section 08) and Sequences & Series (Section 05). Continue practicing with the task file for homework!