

Session 08-03 - Cost Analysis & Pricing Decisions

Section 08: Financial Mathematics

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Entry Quiz - 10 Minutes

Quick Review from Session 08-02

Test your understanding of Annuities

1. Calculate the future value of saving 250/month for 15 years at 4% annual interest.
2. Find the monthly payment for a 30,000 loan at 5.4% for 4 years.
3. After 12 payments on a 15,000 loan at 6% annual for 3 years, what is the outstanding balance?
4. What is the present value of receiving 1,500/month for 10 years at 6% annual?

Learning Objectives

What You'll Master Today

- Understand cost function components: fixed vs. variable costs
- Calculate variable cost per unit $k_v(x)$
- Find the short-term lower limit price (Kurzfristige Preisuntergrenze)
- Find the long-term lower limit price (Langfristige Preisuntergrenze)
- Make pricing and production decisions using cost analysis

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 Exam-Critical Topic

These pricing concepts appear frequently on the Feststellungsprüfung!

Part A: Cost Function Review

Components of Total Cost

Every business has two types of costs:

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! Cost Components

Fixed Costs (K_f or FC): Costs that don't change with production quantity

- Rent, insurance, salaries, equipment depreciation

Variable Costs ($K_v(x)$ or $VC(x)$): Costs that change with quantity produced

- Raw materials, energy, direct labor, packaging

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$$K(x) = K_f + K_v(x)$$

Total Cost Function Examples

Common cost function forms:

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Linear: $K(x) = 500 + 3x$

- Fixed cost: 500
- Variable cost per unit: 3 (constant)

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Quadratic: $K(x) = 400 + 10x + 0.02x^2$

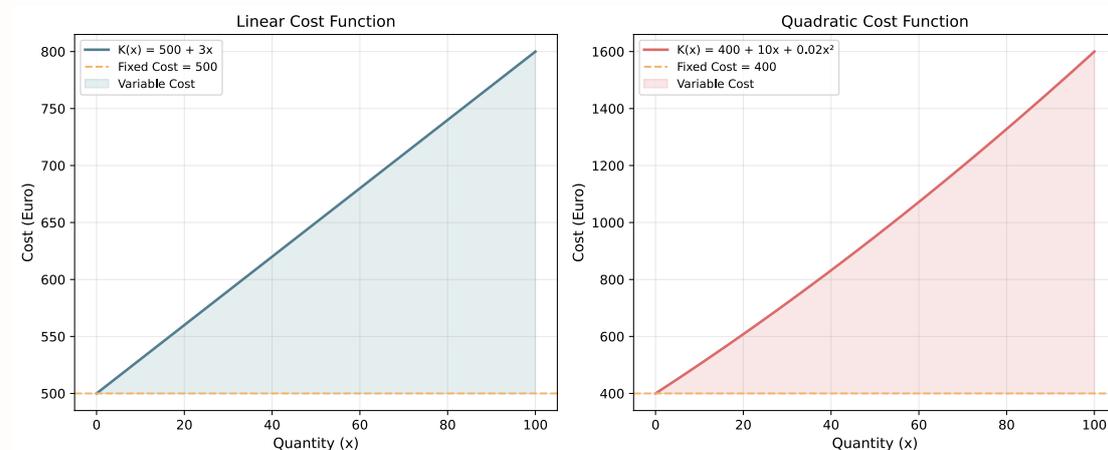
- Fixed cost: 400
- Variable costs increase with quantity

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Cubic: $K(x) = 1000 + 50x - 0.5x^2 + 0.01x^3$

- Most realistic: economies then diseconomies of scale

Visualizing Cost Functions



Part B: Average Cost Functions

Average Total Cost

How much does each unit cost on average?

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! Average Cost (Durchschnittskosten)

$$\bar{K}(x) = \frac{K(x)}{x} = \frac{K_f + K_v(x)}{x} = \frac{K_f}{x} + \frac{K_v(x)}{x}$$

...

This can be written as:

$$\bar{K}(x) = \underbrace{\frac{K_f}{x}}_{\text{Avg. Fixed Cost}} + \underbrace{\frac{K_v(x)}{x}}_{\text{Avg. Variable Cost}}$$

Variable Cost Per Unit

! Variable Cost Per Unit (Stückkosten variabel)

$$k_v(x) = \frac{K_v(x)}{x} = \frac{K(x) - K_f}{x}$$

This is the average variable cost - what each unit costs in variable expenses alone.

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Example: $K(x) = 500 + 10x + 0.01x^2$

$$k_v(x) = \frac{(500 + 10x + 0.01x^2) - 500}{x} = \frac{10x + 0.01x^2}{x} = 10 + 0.01x$$

Why Variable Cost Per Unit Matters

Critical insight for pricing decisions:

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- If price > average total cost: Profit
- If price < average total cost but > variable cost per unit: Loss, but covering some fixed costs
- If price < variable cost per unit: Should NOT produce - losing money on every unit!

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 Warning

The variable cost per unit is the absolute minimum you should charge!

Part C: Short-Term Lower Limit Price

What is the Short-Term Lower Limit?

 Short-Term Lower Limit Price (Kurzfristige Preisuntergrenze)

The minimum price at which a company should continue production in the short term.

$$p_{min,short} = \min\{k_v(x)\} = \text{minimum variable cost per unit}$$

At this price, the company covers its variable costs but not fixed costs.

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When to use:

- Fixed costs are “sunk” (already paid/committed)
- Short-term decision: continue or stop production?

Finding the Minimum Variable Cost Per Unit

Method: Use calculus to find the minimum of $k_v(x)$

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Step 1: Calculate $k_v(x) = \frac{K(x) - K_f}{x}$

Step 2: Find $k_{v'}(x)$ and set equal to zero

Step 3: Solve for x

Step 4: Calculate $k_v(x)$ at this value

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 Tip

Alternatively, use: $k_{v'}(x) = \frac{K'(x) \cdot x - (K(x) - K_f)}{x^2}$

Setting this to zero: $K'(x) \cdot x = K(x) - K_f$

Example: Finding Short-Term Lower Limit

Given: $K(x) = 800 + 20x - 0.2x^2 + 0.002x^3$

Step 1: Variable cost function

$$K_v(x) = 20x - 0.2x^2 + 0.002x^3$$

Step 2: Variable cost per unit

$$k_v(x) = \frac{20x - 0.2x^2 + 0.002x^3}{x} = 20 - 0.2x + 0.002x^2$$

Example (continued)

Step 3: Find minimum

$$k_{v'}(x) = -0.2 + 0.004x = 0$$

$$x = 50$$

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Step 4: Calculate minimum variable cost per unit

$$k_v(50) = 20 - 0.2(50) + 0.002(50)^2 = 20 - 10 + 5 = 15$$

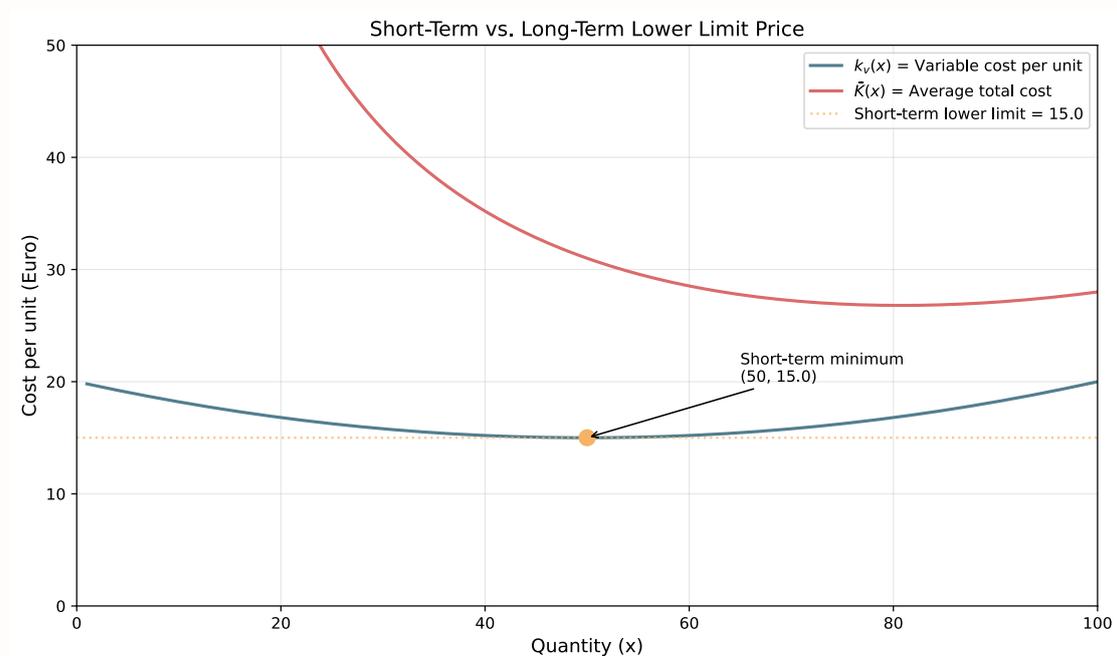
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! Important

Short-term lower limit price = 15 Euro per unit

The company should not sell below 15 Euro, even in the short term!

Graphical Representation



Break - 10 Minutes

Part D: Long-Term Lower Limit Price

Long-Term Lower Limit Price

! Long-Term Lower Limit Price (Langfristige Preisuntergrenze)

The minimum price for sustainable long-term production.

$$p_{min, long} = \min\{\bar{K}(x)\} = \text{minimum average total cost}$$

At this price, the company covers ALL costs (both fixed and variable).

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When to use:

- Long-term strategic pricing decisions
- Deciding whether to enter/exit a market

Finding the Long-Term Lower Limit

Same method, but for average TOTAL cost:

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Step 1: Calculate $\bar{K}(x) = \frac{K(x)}{x}$

Step 2: Find $\bar{K}'(x)$ and set equal to zero

Step 3: Solve for x

Step 4: Calculate $\bar{K}(x)$ at this value

Example: Long-Term Lower Limit

Same cost function: $K(x) = 800 + 20x - 0.2x^2 + 0.002x^3$

Step 1: Average total cost

$$\bar{K}(x) = \frac{800 + 20x - 0.2x^2 + 0.002x^3}{x} = \frac{800}{x} + 20 - 0.2x + 0.002x^2$$

...

Step 2: Find minimum

$$\bar{K}'(x) = -\frac{800}{x^2} - 0.2 + 0.004x = 0$$

...

Multiply by x^2 :

$$-800 - 0.2x^2 + 0.004x^3 = 0$$

$$0.004x^3 - 0.2x^2 - 800 = 0$$

Example (continued)

Solving $0.004x^3 - 0.2x^2 - 800 = 0$ (or $x^3 - 50x^2 - 200000 = 0$):

Using numerical methods or calculator: $x \approx 76.4$

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Step 4: Calculate minimum average cost

$$\bar{K}(76.4) = \frac{800}{76.4} + 20 - 0.2(76.4) + 0.002(76.4)^2 \approx 17.44$$

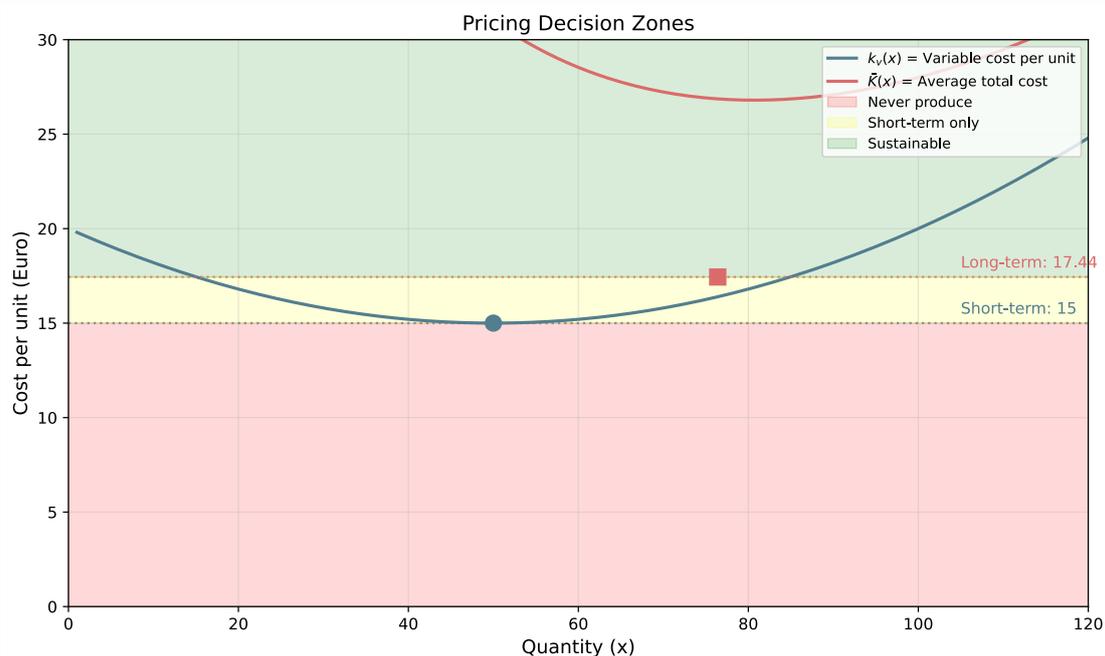
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! Important

Long-term lower limit price = 17.44 Euro per unit

For sustainable production, the company must charge at least 17.44 Euro!

Comparing Short-Term and Long-Term



Part E: Business Interpretation

Pricing Decision Framework

! Three Pricing Zones

Price Level	Decision
$p < k_v^{min}$	STOP production - losing on every unit
$k_v^{min} \leq p < \bar{K}^{min}$	Continue short-term - covers variable costs, contributes to fixed
$p \geq \bar{K}^{min}$	Sustainable - covers all costs

When to Use Each Limit

Short-term lower limit price:

- Factory/equipment already exists
- Fixed costs are committed (“sunk”)
- Deciding whether to accept a special order
- Economic downturn - better to produce and lose less than to stop

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Long-term lower limit price:

- Deciding whether to enter a new market
- Setting regular prices for new products
- Strategic planning and investment decisions

Example: Special Order Decision

A company has excess capacity. A customer offers to buy 100 units at 16 Euro each.

Using our cost function with:

- Short-term lower limit: 15 Euro
- Long-term lower limit: 17.44 Euro

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Analysis:

- $16 > 15$ (short-term limit) ✓
- $16 < 17.44$ (long-term limit)

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💡 Tip

Decision: Accept the order!

While $16 < 17.44$ means we're not covering all costs, each unit sold at 16 contributes 1 Euro toward fixed costs. Better than nothing!

Contribution Margin

! Contribution Margin (Deckungsbeitrag)

$$\text{Contribution per unit} = p - k_v(x)$$

This is how much each unit "contributes" toward covering fixed costs.

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At optimal production level ($x = 50$):

- Price: 16 Euro
- Variable cost per unit: 15 Euro
- Contribution margin: $16 - 15 = 1$ Euro per unit

For 100 units: 100 Euro contribution toward fixed costs!

Guided Practice - 15 Minutes

Practice Problems

Work in pairs

Problem 1: Given $K(x) = 1200 + 30x + 0.1x^2$

- Find the variable cost per unit function $k_v(x)$
- Find the short-term lower limit price
- Find the long-term lower limit price

Problem 2: A company has cost function $K(x) = 500 + 15x$

- What is the short-term lower limit price?
- What is the long-term lower limit price at production of 100 units?
- Should they accept an order for 50 units at 18 Euro each?

Part F: Exam-Style Problems

Typical Exam Problem Structure

FSP exam problems often combine these elements:

1. Given a cost function $K(x)$
2. Find the break-even point(s) given price p

3. Find the profit-maximizing quantity
4. Find the short-term/long-term lower limit price
5. Interpret the results economically

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Warning

Make sure you can distinguish between these questions:

- “Kurzfristige Preisuntergrenze” = Short-term = min of $k_v(x)$
- “Langfristige Preisuntergrenze” = Long-term = min of $\bar{K}(x)$

Summary of Key Formulas

Essential Formulas for Cost Analysis

Concept	Formula
Total Cost	$K(x) = K_f + K_v(x)$
Variable Cost per Unit	$k_v(x) = \frac{K(x) - K_f}{x}$
Average Total Cost	$\bar{K}(x) = \frac{K(x)}{x}$
Short-term Limit	$\min\{k_v(x)\}$
Long-term Limit	$\min\{\bar{K}(x)\}$
Contribution Margin	$p - k_v(x)$

Wrap-Up & Key Takeaways

Today’s Essential Concepts

- Variable cost per unit $k_v(x)$ = variable costs divided by quantity
- Short-term lower limit = minimum variable cost per unit
- Long-term lower limit = minimum average total cost
- Short-term limit < Long-term limit always (fixed costs!)
- Between the limits: Produce short-term, but not sustainable long-term

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Section Complete!

You’ve completed Section 08: Financial Mathematics. Review all formulas before the exam!

Homework Assignment

Tasks 08-03

- Calculate variable cost per unit from various cost functions
- Find short-term and long-term lower limit prices
- Make production decisions based on price comparisons
- Interpret contribution margins in business contexts

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Warning

These pricing decisions are frequently tested! Practice until you can solve them confidently.