Session 03-01 - Function Concepts & Business Modeling

Section 03: Functions as Business Models

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Welcome to Functions!

Introduction

- From equations to functions: A powerful generalization
  - We've solved equations; now we'll model relationships
- Functions as business tools
  - Cost, revenue, and profit modeling
- Mathematical used to model real-world applications
  - Essential for business decision-making

Entry Quiz - 10 Minutes

Quick Review from Section 02

Work individually, then we compare together

a)
$$\begin{cases} 2x + 3y = 18 \\ x - y = 1 \end{cases}$$

b) A company's costs increase exponentially according to $C = 1000 \cdot 1.05^t$. After how many years will costs double?

c) Solve for x: $\log_2(x + 3) + \log_2(x - 1) = 3$

d) Translate: "The profit equals revenue minus costs, where revenue is 50 euros per unit and costs include a fixed cost of 1000 euros plus 30 euros per unit."

Section 02 Review

Your open questions

Ask your questions about the past sections!

- Is there something you are not feeling comfortable with?
- Has there been a task in the exam you found hard?
- Is there any topic you would like to have repeated?

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Tip

This is your chance to have something repeated!

From Equations to Functions

What is a Function?

A function is a rule that assigns to each input exactly one output

- Equation perspective: $y = 2x + 5$ (a relationship)
  - We've been solving these for specific values
- Function perspective: $f(x) = 2x + 5$ (a machine)
  - Input any x, get exactly one output $f(x)$
- Business perspective: A model of cause and effect
  - Input: production quantity $\longrightarrow$ Output: total cost

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Question: Does this make sense for you?

Function Notation

Symbolic language for modeling

$f(x) = 2x^2 - 3x + 1$

- f is the function name (like naming a business model)
- x is the input variable (independent variable)
- $f(x)$ is the output value (dependent variable)

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Tip

Read as: "f of x equals…"

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Note

To find $f(3)$: $f(3) = 2(3)^2 - 3(3) + 1 = 18 - 9 + 1 = 10$

Multiple Function Names

Different functions model different business aspects

- Cost function: $C(x)$ = total cost for x units
- Revenue function: $R(x)$ = total revenue from x units

- Profit function: $P(x) = R(x) - C(x)$
- Demand function: $D(p)$ = quantity demanded at price p

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> **ℹ Note**
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> So far not too difficult, right?

Business Example

A bakery has:

- Fixed costs: 500€ per day
- Variable costs: 2€ per pastry
- Selling price: 5€ per pastry

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Question: What is the cost, the revenue and the profit function?

Quick Practice - 10 Minutes

Function Evaluation Practice

Work individually for 5 minutes, then we discuss

Given the functions:

- $f(x) = 3x^2 - 2x + 1$
- $g(x) = \frac{x+4}{x-2}$

Calculate:

a) $f(0)$, $f(2)$, and $f(-3)$
b) $g(5)$ and $g(0)$

Break - 10 Minutes

Domain and Range

Understanding Domain

The domain is the set of all possible input values

- Mathematical restrictions:
  - Cannot divide by zero
  - Cannot take square root of negative (in real numbers)
  - Logarithm requires positive argument
- Business restrictions:
  - Cannot produce negative quantities

- Limited production capacity
- Budget constraints

Domain Examples

Rational Function

For $f(x) = \frac{1}{x-3}$:

- Denominator cannot be zero
- $x - 3 \neq 0$
- $x \neq 3$
- Domain: $\mathbb{R} \setminus \{3\}$ or $(-\infty, 3) \cup (3, \infty)$

Square Root

For $g(x) = \sqrt{2x+6}$:

- Argument must be non-negative
- $2x + 6 \geq 0$
- $x \geq -3$
- Domain: $[-3, \infty)$

Business Context

Production function $P(x) = 100\sqrt{x}$ where x is hours of labor:

- Mathematical: $x \geq 0$ (square root)
- Practical: $0 \leq x \leq 24$ (hours per day)
- Domain: $[0, 24]$

Understanding Range

The range is the set of all possible output values

- What values can the function actually produce?
- Often harder to find than domain
- Depends on the function's behavior
- Critical for understanding business limitations

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> Tip
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> Range = Output!

Finding Range Examples

Quadratic

For $f(x) = x^2 + 2$:

- When $x = 0$: $f(0) = 2$
- Values grow larger as $|x|$ increases

- Range: $[2, \infty)$
- Note, we'll learn to find exact minimum points soon!

Rational

For $g(x) = \frac{1}{x}$ with domain $x \neq 0$:

- Can be any value except 0
- Range: $\mathbb{R} \setminus \{0\}$

Business Context

Monthly membership revenue $R(x) = 50x$ where x is number of members:

- Minimum: $R(0) = 0$ (no members)
- Increases linearly without bound
- Range: $[0, \infty)$
- Practical limit depends on capacity

Function Representations

Four Ways to Represent Functions

Each representation offers unique insights

1. Verbal: "Base costs of 100 which increase by 3 for each additional unit"
2. Algebraic: $C(x) = 100 + 3x$
3. Numerical: Table of values
4. Graphical: Visual representation

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Question: How would you represent this function as table and as graph?

Example: Mobile Phone Plan

Scenario: Mobile plan costs 20€ base fee plus 0.10€ per minute.

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Question: How would you represent this as a function, as a table and as a graph?

Graphical Representation

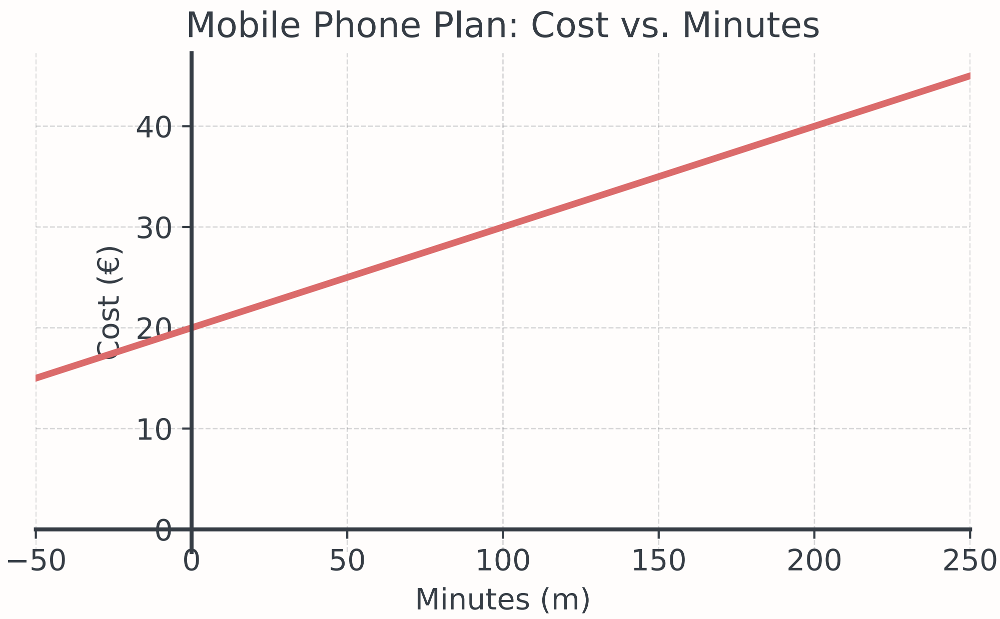

Figure 1: $C(m) = 20 + 0.10m$

The Vertical Line Test

A graph only represents a function if every possible vertical line intersects it at most once

- Why? Each input must have exactly one output
- Pass: Linear, quadratic, exponential graphs
- Fail: Circles, sideways parabolas
- Business implication: No ambiguity in predictions

Is this a function?

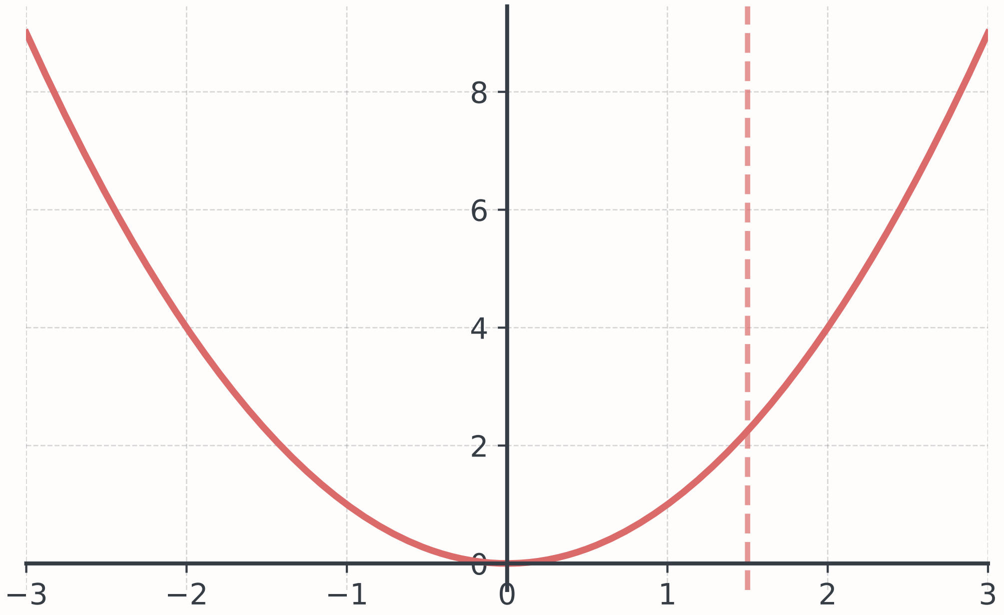

Figure 2: $y = x^2$

What about this?

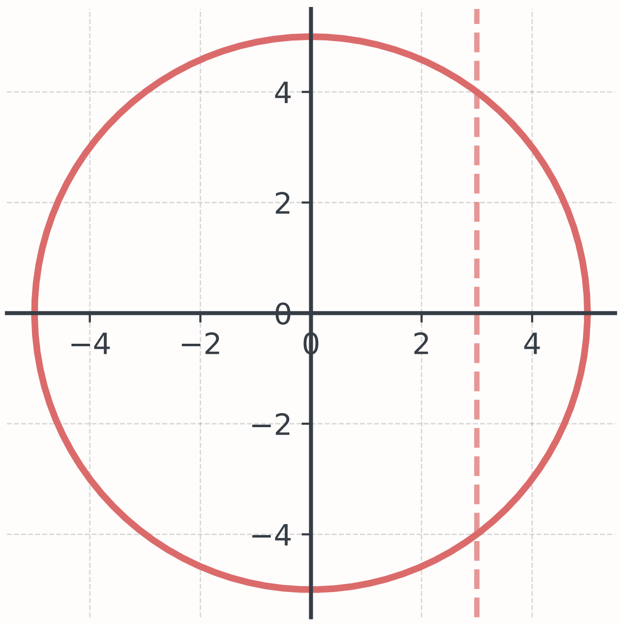

Figure 3: $x^2 + y^2 = 25$

Guided Practice - 25 Minutes

Individual Exercise Block

Work alone for 15 minutes

a) Find the domain of: $f(x) = \frac{x+2}{x^2-4}$

b) Cost function is $C(x) = 1500 + 25x$ and revenue is $R(x) = 40x$

   a. Find the profit function $P(x)$
   b. What is the break-even point?
   c. What is a reasonable domain for this model?

c) Given the function $g(x) = 2x + 8$:

   a. Find the domain and range
   b. Find where $g(x) = 20$

Business Applications

Cost Functions in Detail

Understanding the structure of business costs

$$C(x) = \text{Fixed Costs} + \text{Variable Costs}$$

- Fixed Costs (FC): Rent, insurance, salaries
  - Independent of production level
- Variable Costs (VC): Materials, hourly wages, utilities
  - Proportional to production
- Total Cost: $C(x) = FC + VC \cdot x$
- Average Cost: $AC(x) = \frac{C(x)}{x}$

Revenue and Demand

The relationship between price and quantity

- Simple model: $R(x) = p \cdot x$ (fixed price)
  - Linear revenue function
- Reality: Price often depends on quantity sold
  - Higher supply $\longrightarrow$ lower price
  - This creates more complex models
- Preview: In Session 03-03, we'll explore quadratic models
  - These allow us to find optimal quantities
  - Essential for maximizing profit

Combining Functions

Example: Concert Tickets

- Fixed price: 40€ per ticket
- Fixed costs: 10,000€
- Variable costs: 10€ per ticket

- Revenue: $R(x) = 40x$ (linear function)
- Cost: $C(x) = 10000 + 10x$ (linear function)
- Profit: $P(x) = R(x) - C(x) = 40x - (10000 + 10x)$

- Simplified: $P(x) = 30x - 10000$ (also linear!)
- Break-even: When $P(x) = 0$, so $x = 333.33 \rightarrow$ need 334 tickets

Collaborative Problem-Solving

Group Activity: Startup Analysis

The Scenario

A startup produces custom phone cases:

- Fixed monthly costs: 3,000€ (rent, equipment, insurance)
- Material cost per case: 8€
- Labor cost per case: 7€
- They plan to sell at a fixed price of 35€ per case

Your Tasks

Work in groups of if you like

a) Develop the cost function $C(x)$
b) Develop the revenue function $R(x)$
c) Find the profit function $P(x)$
d) Determine the break-even quantity (where $P(x) = 0$)
e) If they can produce maximum 500 cases per month, what's their maximum possible profit?

Coffee Break - 15 Minutes

Practice Tasks

Identifying Functions (x)

Work individually and then we compare.

Determine whether each relation represents a function. If it is not a function, explain why using the vertical line test concept.

a) $y = 4x - 7$
b) $x^2 + y^2 = 25$
c) $y = |x - 2|$
d) $x = y^2 + 1$

Fitness Center Membership Model

A center has collected data on how membership varies with price.

Monthly Fee (€)	Number of Members
30	400

Monthly Fee (€)	Number of Members
40	350
50	300
60	250

The fitness center has:

- Fixed monthly costs: €15,000
- Variable costs: €5 per member

Your Tasks

Work in groups for 20 minutes

a) Show that this data represents a function with monthly fee as input and number of members as output
b) Write the function $N(p)$ where p is the monthly fee (assuming a linear relationship)
c) Find a reasonable domain for this business context
d) Write the profit function $P(p)$ in terms of the monthly fee p

Wrap-Up

Key Takeaways

- Functions model relationships between variables
- Domain and range define boundaries of our models
- Business applications require multiple functions working together
- The same relationship can be represented in multiple ways
- Real-world constraints affect mathematical models

Final Assessment

5 minutes - Individual work

A local gym has fixed costs of 5000€ per month and variable costs of 10€ per member. They charge 40€ per member per month.

a) Write the cost function $C(x)$ where x is the number of members
b) Write the revenue function $R(x)$
c) How many members are needed to break even?

Next Session Preview

03-02: Linear Functions & Economic Applications

- Deep dive into linear functions
- Supply and demand curves
- Market equilibrium
- Linear regression basics

- Cost-volume-profit analysis

> Tip
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> Homework Assignment: Complete Tasks 03-01!