# Session 02-01 - Equations & Inequalities

## Section 02: Equations & Problem-Solving Strategies

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# Entry Quiz

## Quick Review from Section 01

10 minutes - individual work, then we review

a) Factor completely: $x^6 - 7x^3 - 8$

b) Simplify: $\frac{(3x^{-2}y^3)^{-2} \cdot (2x^3y^{-1})^3}{6x^{-4}y^2}$

c) If $2^{x+1} + 2^x = 24$, find $x$

d) Rationalize and simplify: $\frac{3}{\sqrt{5}+\sqrt{3}} - \frac{2}{\sqrt{5}-\sqrt{3}}$

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> **Tip**
>
> Present your solutions and we review together!

# Homework Presentations

## Solutions Showcase

20 minutes - presentations and discussion

- Discuss your most challenging problem from Tasks 01-06
- Share your problem-solving approach
- Show potential alternative methods
- Ask questions about problems you found difficult

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> **Tip**
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> Remember: Discussing tasks helps solidify your own understanding!

# Key Concept Review

## The IDEA Method

A method to help you assess tasks

- Identify: What type of problem are we solving?
- Develop: Create a plan using appropriate methods
- Execute: Carry out the solution carefully
- Assess: Check your answer makes sense

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> **Note**
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> Today we apply IDEA to translating word problems into equations and inequalities!

# Mathematical Language

## Translation Fundamentals

Converting words to mathematical expressions

| English Phrase | Symbol | Example |
|---|---|---|
| "is", "equals", "is equal to" | = | "The cost is €50" $\longrightarrow C = 50$ |
| "less than", "fewer than" | < | "x is less than 10" $\longrightarrow x < 10$ |
| "at least", "no less than" | ≥ | "at least 5 units" $\longrightarrow x \geq 5$ |
| "at most", "no more than" | ≤ | "at most 100" $\longrightarrow x \leq 100$ |
| "increased by", "plus" | + | "price increased by €5" $\longrightarrow p + 5$ |
| "decreased by", "minus" | - | "reduced by 20%" $\longrightarrow x - 0.2x$ |
| "of", "times" | × | "30% of sales" $\longrightarrow 0.3S$ |

## Business Vocabulary Essentials

Key terms you'll encounter frequently

- Revenue (R): Total income = Price × Quantity
- Cost (C): Fixed costs + Variable costs
- Profit (P): Revenue - Cost = R - C
- Break-even: When Revenue = Cost (Profit = 0)
- Margin: Profit as percentage of revenue
- Markup: Increase from cost to selling price

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> Tip
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> Always define your variables clearly before translating!

### Practice IDEA with Tasks

Lets practice this! Try these on your own

Translate each phrase into an equation and solve:

a) “Seven more than twice a number equals 31”

b) “The quotient of a number and 4, decreased by 3, is 12”

c) “40% of a number increased by 25 equals the number itself”

## Break - 10 Minutes

### Recap: Solving Multi-Step Equations

A systematic approach

1. Clear fractions: Multiply by LCD
2. Expand: Remove parentheses using distributive property
3. Collect terms: Variables on one side, constants on other
4. Isolate variable: Divide by coefficient
5. Verify: Substitute back into original equation

### Example: Equation with Fractions

Let’s work through this together

Solve: $\frac{2x-1}{3} + \frac{x+2}{4} = 5$

- Step 1: Find LCD $\longrightarrow$ LCD = 12
- Step 2: Clear fractions $\longrightarrow 12 \cdot \frac{2x-1}{3} + 12 \cdot \frac{x+2}{4} = 12 \cdot 5$
- Step 3: Simplify $\longrightarrow 4(2x-1) + 3(x+2) = 60$
- Step 4: Expand $\longrightarrow 8x - 4 + 3x + 6 = 60$
- Step 5: Combine $\longrightarrow 11x + 2 = 60$
- Step 6: Solve $\longrightarrow 11x = 58$, so $x = \frac{58}{11}$

### Recap: Inequalities

When things aren’t necessaryly equal

- When multiplying or dividing by negative number, flip the sign!
  - Example: $-2x > 6$
  - Divide by –2: $x < -3$ (sign flipped!)
  - Why? Because the number line reverses!
- Inequalities are used to restrict the range of a variable
- Often Used to bound the solution space in business applications

### Example: Business Application

Profit constraints in action

A company has costs $C = 5000 + 20x$ and revenue $R = 50x$.

How many units must they sell to make at least €4000 profit?

- Set up: Profit = Revenue - Cost ≥ 4000
- Equation: $50x - (5000 + 20x) \geq 4000$
- Simplify: $30x - 5000 \geq 4000$
- Solve: $30x \geq 9000$, so $x \geq 300$
- Answer: Must sell at least 300 units

## Practice

### Individual Exercises

Work independently, then we'll discuss

a) To equation: "Three times a number decreased by 7 equals 14"

b) Solve: $3(2x - 4) = 2(x + 5)$

c) Solve the inequality: $-3x + 7 < 16$

d) A taxi charges €3.50 base fare plus €1.20 per km. If a ride costs €15.50, how far was it?

e) A store offers 30% discount. After discount, an item costs €42. What was the original price?

## Application & Extension

### Break-Even Analysis

Where total revenue equals total cost (profit = 0)

A coffee shop has fixed costs of €2,000/month (rent, utilities), variable cost of €1.50 per coffee and a selling price of €3.50 per coffee. How many coffees for break-even?

- Let $x$ = number of coffees
- Cost: $C = 2000 + 1.50x$
- Revenue: $R = 3.50x$
- Break-even: $3.50x = 2000 + 1.50x$
- Solve: $2x = 2000$, so $x = 1000$ coffees

### Mixture Problems

Combining different concentrations or values

An investor has €10,000 to split between bonds (4% return) and stocks (9% return). To earn €650 annually, how much in each?

- Let $x$ = amount in bonds
- Then $10000 - x$ = amount in stocks
- Income equation: $0.04x + 0.09(10000 - x) = 650$
- Simplify: $0.04x + 900 - 0.09x = 650$
- Solve: $-0.05x = -250$, so $x = 5000$
- Answer: €5,000 in bonds, €5,000 in stocks

# Coffee Break - 15 Minutes

# Collaborative Problem-Solving

## Group Task

Work in groups on the following problem

A company produces two products:

- Product A: Costs €15 to make, sells for €25
- Product B: Costs €20 to make, sells for €35
- Fixed costs: €5,000/month
- Production capacity: 500 units total
- Must produce at least 100 of each product

## The tasks

Work in groups on the following problem

a) Set up the profit equation
b) Find the break-even point if producing equal quantities
c) What mix maximizes profit?

# Wrap-up & Synthesis

## Key Takeaways

Essential skills from today

- Translation from words to equations is systematic
- Multi-step equations require organized approach
- Inequalities have special rules (flip when multiplying by negative!)
- Business problems often involve setting up profit/cost equations
- Break-even analysis is fundamental to business planning

## Common Pitfalls to Avoid

Watch out for these!

- Forgetting to flip inequality signs
- Misinterpreting “less than” in word problems
- Not checking solutions in original equation

- Mixing up revenue and profit
- Forgetting units in final answers

## Final Assessment

Individual work

A small business has monthly costs of €3,000 plus €12 per unit produced. They sell each unit for €20.

a) Write the profit equation
b) How many units for break-even?
c) How many units for €2,000 profit?

## Next Session Preview

Session 02-02: Systems of Equations

- Solving systems by substitution and elimination
- Business applications with multiple constraints
- Introduction to linear programming

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> Preparation Tip
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> Review today's equation-solving techniques - they're the foundation for systems!