# Session 01-06 - Synthesis & Applications

### Section 01: Mathematical Foundations & Algebra

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## Warm-up Quiz

### Quick Skills Check

5 minutes - test your readiness

a) Simplify:  $(2x^3)^2 \cdot x^{-5}$ 

b) Factor:  $x^2 - 5x - 14$ 

c) Evaluate:  $log_2(32)$ 

d) Simplify:  $\sqrt{48}$ 

e) Express in scientific notation: 0.000425

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These cover the core skills - let's review together!

### **Solution Presentations**

#### **Showcase Your Solutions**

25 minutes for comprehensive discussions

- Present your most challenging problem from Tasks 01-05
- Are there any tasks from previous lectures you want to discuss?
- Explain your problem-solving strategy
- Share any connections between topics you discovered

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Today we see how all pieces fit together!

## **Key Concepts**

## Set Theory & Number Systems

**Essential foundations** 

Number hierarchy: \$ I \$

Set operations:

• Union:  $A \cup B$  (all elements in either)

• Intersection:  $A \cap B$  (elements in both)

• Difference:  $A \setminus B$  (in A but not B)



Quick practice:

• If  $A = \{1, 3, 5\}$  and  $B = \{3, 4, 5, 6\}$ 

• Find  $A \cap B$  and  $A \cup B$ 

#### **Exponent Laws**

Your main tools for exponents

Rule	Formula	Quick Example
Product	$a^m \cdot a^n = a^{m+n}$	$x^3 \cdot x^4 = x^7$
Quotient	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^5}{x^2} = x^3$
Power	$\left(a^{m}\right)^{n}=a^{mn}$	$\left(x^3\right)^2 = x^6$
Negative	$a^{-n} = \frac{1}{a^n}$	$x^{-2} = \frac{1}{x^2}$

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Quick practice: Simplify  $\frac{\left(3x^2\right)^3}{9x^4}$ 

## **Factorization Techniques**

Your factoring toolbox

1. Common factor: Always check first!

2. Difference of squares:  $a^2 - b^2 = (a + b)(a - b)$ 

3. Perfect squares:  $a^2 \pm 2ab + b^2 = (a \pm b)^2$ 

4. AC method: For  $ax^2 + bx + c$  when  $a \neq 1$ 

5. Grouping: For 4-term polynomials

6. Cubes:  $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$ 

7. Substitution: Let \$u = \$ common expression

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Quick practice: Factor  $x^4-16$ 

#### Radicals & Logarithms

Advanced operations

Radicals:

- Simplify by extracting perfect powers
- Rationalize using conjugates:  $\left(a+\sqrt{b}\right)\!\left(a-\sqrt{b}\right)=a^2-b$

Logarithms:

- $\log_a(xy) = \log_a(x) + \log_a(y)$
- $\log_a(x^n) = n \log_a(x)$
- If  $a^x = b$ , then  $x = \log_a(b)$



Quick practice: Solve  $2^x = 10$ 

#### Practice Block 1 - Fundamentals

Individual Exercise

10 minutes - repeat the basics

- 1. Simplify:  $\frac{x^3 \cdot x^{-5}}{x^{-3}}$
- 2. Factor completely:  $3x^2 27$
- 3. Rationalize:  $\frac{6}{\sqrt{3}}$
- 4. Solve: |3x 6| = 9
- 5. Evaluate:  $\log_3(81) \log_3(3)$
- 6. Express in scientific notation: 45,600,000

Break - 10 Minutes

Practice Block 2 - Fundamentals

Pair Exercise

15 minutes - work together

1. Factor using substitution:  $x^4 - 5x^2 + 4$ 

2. Simplify:  $\sqrt{75} + \sqrt{48} - \sqrt{27}$ 

3. If  $2^x = 3$  and  $3^y = 5$ , find  $2^{xy}$ 

4. Rationalize:  $\frac{4}{3-\sqrt{5}}$ 

5. Factor by grouping:  $x^3 + 2x^2 - 9x - 18$ 

## Mixed Techniques

### Mixed Technique Problems

Combine multiple methods

1. Factor:  $8x^3 - 27y^3$ 

2. Simplify:  $\frac{x^3-8}{x^2-4} \cdot \frac{x+2}{x^2+2x+4}$ 

3. If |2x - 5| < 7, find the solution interval

4. Rationalize:  $\frac{2}{\sqrt{6}-\sqrt{2}}$ 

## **Business Applications**

#### Financial Growth Problems

Real-world application with compound interest

An investment of €5,000 earns 8% annual interest compounded yearly.

a) Write the formula for the amount after t years

b) Calculate the value after 10 years

c) When will it triple in value?

## Cost-Profit Analysis

Combining algebraic techniques for Manufacturing

A company produces x units with:

• Cost:  $C = 2x^2 + 100x + 5000$ 

• Revenue:  $R = 500x - 3x^2$ 

Your tasks:

a) How can you compute the profit?

b) Factor the profit completely

## Coffee Break - 15 Minutes

## Another Block

Synthesis Problems

Comprehensive practice

1. Solve:  $\log_3(x^2 - 8) = 2$ 

2. Factor:  $x^3 + 3x^2 - x - 3$ 

3. Solve:  $\log_2(x+5) = 3$ 

4. Factor:  $x^3 - 27$ 

5. Simplify:  $\frac{2\sqrt{12}+3\sqrt{27}}{\sqrt{3}}$ 

#### Challenge Problems

Test your skills by solving these in pairs

1. If  $3^x + 3^{-x} = 4$ , find  $9^x + 9^{-x}$ 

2. Factor completely:  $x^6 - 7x^3 - 8$ 

3. A bacteria culture triples every 4 hours. Starting with 500 bacteria:

• When will there be 1 million bacteria?

• Express the population after 1 day in scientific notation

## Wrap-up

#### **Key Skills Mastery**

Your Section 1 foundation is complete!

• Can you factor any polynomial type we've studied?

• Can you apply all exponent laws confidently?

• Can you simplify complex radical expressions?

• Can you solve logarithmic equations?

• Can you work with absolute values?

• Can you convert to/from scientific notation?

• Most importantly: Can you combine techniques to solve complex problems?

#### Study Strategy Going Forward

Success in mathematics requires consistent practice

Daily (10-15 minutes):

Review one concept from past sections

• Solve 2-3 smaller problems

Check your solutions carefully

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Do this in addition to our lecture and the problems we solve together here.

### **Moving Forward**

You've built a strong foundation!

Your achievements:

- Mastered algebraic foundations
- Learned powerful problem-solving techniques
- Built confidence with complex problems
- Prepared for advanced mathematics

Next: Section 2 - Functions and Calculus Basics

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! Important

Keep practicing Section 1 skills alongside new material - they remain essential!

## **Questions & Reflection**

## Open Discussion

Share your thoughts

- What topic needs more review?
- Which connections were most helpful?
- Any remaining questions?
- How do you feel about the material?

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Mathematics builds on itself - your foundation is now solid!