Session 01-02 - Language, Sets, and Number Systems

Section 01: Mathematical Foundations & Algebra

Dr. Nikolai Heinrichs & Dr. Tobias Vlćek

Your Confidence

Your Confidence in Topics

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Error: The file was not found at ../data/tally_start_01.csv
Please make sure the 'data-folder' directory and 'data.csv' file exist.
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> **Note**
>
> That's a good foundation for this course! :)

Entry Quiz

Quick Morning Check

Complete on paper - we'll review together

a) Calculate: $\frac{2}{3} + \frac{3}{4}$
b) Simplify: $x^2 \cdot x^3$
c) What is 15% of 240?
d) Solve: $|x| = 5$

. . .

> **Tip**
>
> Ready? Let's see how you did!

Number Systems

The Number Hierarchy

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{I} \subset \mathbb{R}$$

- $\mathbb{N} = \{1, 2, 3, \ldots\}$ Natural numbers
- $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ Integers

- $\mathbb{Q} = \left\{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\right\}$ Rationals
- $\mathbb{I}$ = Numbers that cannot be expressed as fractions Irrationals
- $\mathbb{R}$ = All points on the number line Reals

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Note

Some books include 0 in $\mathbb{N}$, denoted $\mathbb{N}_0$. For this course, we define $\mathbb{N} = \{\mathbf{1, 2, 3, \ldots}\}$. The set including zero is denoted $\mathbb{N}_0$.

Set Theory Basics

What is a Set?

A set is a well-defined collection of distinct objects.

Notation:

- Roster: $A = \{1, 2, 3, 4, 5\}$
- Set-builder: $B = \{x \in \mathbb{N} : x < 6\}$
- Interval: $C = [0, 1] = \{x \in \mathbb{R} : 0 \leq x \leq 1\}$

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Common Mistake

$\{1, 2, 2, 3\} = \{1, 2, 3\}$ — Sets contain only distinct elements!

Interval Notation

Interval notation uses brackets and parentheses to show whether endpoints are included or excluded:

Closed Intervals: Both endpoints included

- $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$
- Example: $[1, 5]$ includes both 1 and 5

Open Intervals: Both endpoints excluded

- $(a, b) = \{x \in \mathbb{R} : a < x < b\}$
- Example: $(1, 5)$ excludes both 1 and 5

Mixed Intervals: One endpoint included, one excluded

- $[a, b) = \{x \in \mathbb{R} : a \leq x < b\}$ (includes a, excludes b)
- $(a, b] = \{x \in \mathbb{R} : a < x \leq b\}$ (excludes a, includes b)

Mathematical Language

Why Mathematical Notation?

Mathematics is a universal language that allows us to:

- Express complex ideas precisely
- Communicate without ambiguity
- Solve problems systematically
- Build logical arguments

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> **i Note**
>
> Clear notation prevents costly misunderstandings in contracts, financial models, and data analysis.

Essential Symbols - Sets

Symbol	Meaning	Example
$\in$	Element of	$3 \in \mathbb{N}$
$\notin$	Not element of	$\pi \notin \mathbb{Q}$
$\subset$	Subset	$\mathbb{N} \subset \mathbb{Z}$
$\subseteq$	Subset or equal	$A \subseteq A$
$\cup$	Union	$A \cup B$
$\cap$	Intersection	$A \cap B$
$\emptyset$	Empty set	$A \cap B = \emptyset$

Essential Symbols - Logic

Symbol	Meaning	Example
$\forall$	For all	$\forall x \in \mathbb{R} : x^2 \geq 0$
$\exists$	There exists	$\exists x \in \mathbb{Z} : x < 0$
$\Rightarrow$	Implies	$x = 2 \Rightarrow x^2 = 4$
$\Leftrightarrow$	If and only if	$x^2 = 4 \Leftrightarrow x = \pm 2$
$\neg$	Not	$\neg(x > 0)$ means $x \leq 0$
$\wedge$	And	$p \wedge q$
$\vee$	Or	$p \vee q$

Let's Practice Reading

Translate to English:

- $\forall x \in \mathbb{R} : x + 0 = x$
  - "For all real numbers x, x plus zero equals x"
- $\exists n \in \mathbb{N} : n > 1000000$
  - "There exists a natural number n greater than one million"
- $x \in A \cap B \Rightarrow x \in A$
  - "If x is in the intersection of A and B, then x is in A"

Individual Exercise 01

Work individually first, then compare with neighbors

Express the following in set notation (choose which notation to use):

a) The set of all even natural numbers
b) The set of all real numbers between –1 and 1 (inclusive)
c) The set of all integers divisible by 3

Break - 10 Minutes

Venn Diagrams

Venn diagrams are visual representations of sets and their relationships.

Components:

- Rectangle: Universal set U
- Circles: Individual sets
- Overlaps: Intersections
- Outside circles: Complements

Example:

- U = {1, 2, 3, 4, 5, 6, 7, 8}
- A = {1, 2, 3, 4}
- B = {3, 4, 5, 6}
- A ∩ B = {3, 4} (overlap)

. . .

> Tip
>
> Venn diagrams help visualize complex set relationships!

Working with Numbers & Sets

Set Operations

Union: $A \cup B$

- All elements in A or B
- Example: $\{1, 2\} \cup \{2, 3\} = \{1, 2, 3\}$

Intersection: $A \cap B$

- All elements in A and B
- Example: $\{1, 2\} \cap \{2, 3\} = \{2\}$

Difference: $A \setminus B$

- Elements in A but not in B
- Example: $\{1, 2, 3\} \setminus \{2, 3\} = \{1\}$

Complement: $\bar{A}$

- All elements not in A
- Requires universal set U

Example: Employee Skills

A company tracks employee skills:

- P = {Python, Java, SQL, R}
- D = {SQL, Excel, Tableau, R}

Lets work together and find the following:

- $P \cup D$ (All skills available)
- $P \cap D$ (Versatile skills)
- $P \setminus D$ (Skills unique to Programmers)
- $D \setminus P$ (Skills unique to Data Analysts)

Classifying Numbers

Example 1

Classify $\frac{22}{7}$

- Can be written as $\frac{p}{q}$ ✓
- Therefore: $\frac{22}{7} \in \mathbb{Q}$
- Also: $\frac{22}{7} \in \mathbb{R}$
- But: $\frac{22}{7} \notin \mathbb{Z}$ (≈ 3.14…)

Example 2

Classify $\sqrt{9}$

- $\sqrt{9} = 3$
- Therefore: $3 \in \mathbb{N}$
- Also: $3 \in \mathbb{Z}, \mathbb{Q}, \mathbb{R}$

Common Mistakes

Be careful! $\pi \approx \frac{22}{7}$ but $\pi \neq \frac{22}{7}$

- π is irrational!
- No fraction exactly equals π

What about the following?

Is $0.\overline{45}$ rational?

Method

Let $x = 0.454545...$

- Multiply by 100: $100x = 45.454545...$
- Subtract original: $100x - x = 45.454545... - 0.454545...$
- Simplify: $99x = 45$
- Solve: $x = \frac{45}{99} = \frac{5}{11}$

Yes! It's rational.

General Rule

For $0.\overline{abc}$ with n repeating digits:

- Multiply by 10^n
- Subtract original
- Solve for x

Group Exercise 01

How is this class structured?

Using set notation and a Venn diagram, find:

a) How many study at least one subject?
b) How many study only Mathematics?

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> Tip
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> We are going to do this one together. Any suggestions on how to start?

Individual Exercise 02

Number Classification

Classify each (list ALL applicable sets):

a) $-\frac{8}{2}$
b) $\sqrt{7}$
c) $0.\overline{55}$

d) $\pi + 1$

Break - 15 Minutes

Properties of Operations

The Big Three Properties

1. Commutative: Order doesn't matter
   - $a + b = b + a$
   - $a \times b = b \times a$
2. Associative: Grouping doesn't matter
   - $(a + b) + c = a + (b + c)$
   - $(a \times b) \times c = a \times (b \times c)$
3. Distributive: Multiplication distributes over addition
   - $a(b + c) = ab + ac$

Business Application

Revenue Calculation

A store sells 3 products:

- Product A: 50 units at €20 each
- Product B: 30 units at €20 each
- Product C: 40 units at €20 each

Method 1

$(50 \times 20) + (30 \times 20) + (40 \times 20)$

Method 2

$20 \times (50 + 30 + 40)$ Distributive property!

Common Mistakes

- Remember: multiplication before addition!

Which Operations Commute?

Operation	Commutative?	Example
Addition	✓ Yes	$3 + 5 = 5 + 3 = 8$
Multiplication	✓ Yes	$3 \times 5 = 5 \times 3 = 15$
Subtraction	✕ No	$5 - 3 \neq 3 - 5$
Division	✕ No	$6 \div 2 \neq 2 \div 6$

Operation	Commutative?	Example
Exponentiation	✕ No	$2^3 \neq 3^2$

...

⚠ Common Mistake

Students somtimes assume all operations commute!

Percentage Calculations

Basic Percentage

Finding x% of a number:

$$\text{Result} = \frac{x}{100} \times \text{Base}$$

- Example: 15% of 240
- Solution: 15% of 240 = $\frac{15}{100} \times 240 = 36$

Percentage Change

Finding the change:

$$\text{Change }\% = \frac{\text{New} - \text{Old}}{\text{Old}} \times 100\%$$

- Example: From €5000 to €5500
- Solution: $\frac{5500-5000}{5000} \times 100\% = 10\%$

Compound Growth

Multiple periods:

$$\text{Final} = \text{Initial} \times (1 + r)^n$$

- Where r = rate (as decimal), n = periods
- Example: €5000 at 10% for 3 years
- Solution: $5000 \times (1.10)^3 = €6655$

Mathematical Logic Basics

Truth Tables for propositions p and q

p	q	$p \wedge q$ (and)	$p \vee q$ (or)	$p \Rightarrow q$ (imp.)
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

. . .

> Understanding Implication ($p \Rightarrow q$)
>
> Think of it as a promise: "If it is raining (p), then I will carry an umbrella (q)." The only way the promise is broken (the statement is False) is if it's raining (p=T) but I don't have my umbrella (q=F).

Soft Introduction to Proofs

A proof is a logical argument that shows a statement is true.

- The goal is to move from what we know (assumptions) to what we want to show (conclusion) using small, logical steps.
- A direct proof is the most common form:
- Assume p is true and show that q must logically follow.

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> Tip
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> No need to worry about this topic too much! We just cover the absolute basics here just for you to know what a proof is.

Practice Time

Group Exercise 02

Working in pairs, determine if these statements are true or false:

a) $\mathbb{Z} \subset \mathbb{Q}$
b) $\sqrt{4} \in \mathbb{N}$
c) $0.333... \in \mathbb{Q}$
d) $\{1, 2\} \subset \{1, 2, 3\}$
e) $\emptyset \subset \mathbb{N}$

. . .

Take 5 minutes, then we'll discuss!

Wrap-up

Key Takeaways

- Mathematical notation is precise and universal
- Venn diagrams visualize set relationships
- Number systems form a hierarchy: $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$
- Repeating decimals are rational numbers
- Operations have specific properties we can exploit

- Percentages and compound growth are essential for business
- Logic helps us reason systematically

For Next Time

Homework: Complete Tasks 01-02

Focus on:

- Set operations practice
- Number classification
- Proving/disproving properties
- One presentation problem

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> **! Important**
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> Entry quiz next session on today's material!