Mini-Mock Exam 04: Advanced Functions

BFP Mathematics Course
Dr. Nikolai Heinrichs & Dr. Tobias Vlćek

Name: ______________________

Reading Time: 10 minutes

Working Time: 90 minutes

Permitted Aids:

- Calculator (non-programmable without graphing capabilities)
- Drawing instruments
- No formula sheets or notes

i Grading Reference

Grade	Points Required	Percentage
1 (Excellent)	45-50	90-100%
2 (Very Good)	39-45	77-90%
3 (Good)	32-39	63-77%
4- (Pass)	23-32	45-63%
5-6 (Fail)	0-23	0-45%

Note: Passing grade requires at least 23 points (45%).

Problem 1: Exponential Growth Model [20 pts. total]

A biotechnology company is developing a new bacterial culture for pharmaceutical production. The bacteria population follows an exponential growth model under controlled conditions.

Part A: Population Analysis

The initial population is 500 bacteria. After 3 hours, the population has grown to 4,000 bacteria.

a) Determine the exponential growth function $P(t) = P_0 \cdot a^t$, where t is time in hours. Show all steps in your calculation. [5 pts.]

b) Calculate the population after 5 hours using your model from part (a). [3 pts.]

c) Determine when the population will reach 32,000 bacteria. Express your answer as an exact value using the growth rate you found. [3 pts.]

Part B: Practical Applications

The bacterial culture produces a valuable enzyme. The production facility has a maximum capacity of 256,000 bacteria.

d) Calculate how long it takes to reach 50% of the maximum capacity (128,000 bacteria). [4 pts.]

e) The doubling time is the period required for the population to double. Using your model, show that the doubling time is exactly 1 hour. [2 pts.]

f) If the bacteria must be harvested when the population is between 100,000 and 200,000 for optimal enzyme yield, determine the time window (in hours) when harvesting should occur. Express your answer using complete sentences. [3 pts.]

Problem 2: Function Transformation and Analysis [20 pts. total]

Consider the base function $f(x) = \sqrt{x}$ and its transformation $g(x) = -2\sqrt{x+4} + 6$.

Part A: Transformation Identification

a) List all transformations applied to $f(x)$ to obtain $g(x)$ in the correct order of application. [3 pts.]

b) Determine the domain and range of $g(x)$. Show your reasoning. [4 pts.]

c) Find the x-intercept and y-intercept of $g(x)$ algebraically. Show all steps. [4 pts.]

Part B: Comparative Analysis

Consider the additional function $h(x) = \log_2(x + 4)$.

d) Determine the point(s) of intersection between $g(x)$ and $h(x)$. [3 pts.]

e) The function $g(x)$ has a maximum value. Determine this maximum value and the x-value where it occurs. Explain your reasoning using complete sentences. [3 pts.]

f) Sketch both functions $g(x)$ and $h(x)$ on the same coordinate system for $x \in [-4, 16]$. Clearly label:
- All intercepts
- Points of intersection
- Asymptotes (if any)
- Key points used for graphing [3 pts.]

Problem 3: Trigonometric Modeling [10 pts. total]

A coastal engineering firm is studying tidal patterns at a harbor to optimize shipping schedules. The water depth $D(t)$ in meters varies sinusoidally with time t in hours after midnight.

Given Information:

- At midnight ($t = 0$), the water depth is 8 meters
- The maximum depth of 14 meters occurs at 6:00 AM ($t = 6$)
- The minimum depth of 2 meters occurs at 6:00 PM ($t = 18$)
- The tidal pattern repeats every 24 hours

Part A: Model Development

a) Determine the amplitude, midline, and period of the tidal function. Show your calculations. [3 pts.]

b) Write the tidal depth function in the form $D(t) = A\sin(B(t - C)) + D$ or $D(t) = A\cos(B(t - C)) + D$. Explain your choice of sine or cosine and justify all parameter values. [4 pts.]

Part B: Practical Applications

Large cargo ships require a minimum depth of 10 meters to safely enter the harbor.

c) Using your model from part (b), determine the time intervals during a 24-hour period when ships can safely enter the harbor. Express your answer in interval notation and using complete sentences. [3 pts.]