Mini-Mock Exam 03: Functions & Business Models

BFP Mathematics Course
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Name: _____

Reading Time: 10 minutes

Working Time: 90 minutes

Permitted Aids:

- Calculator (non-programmable without graphing capabilities)
- Drawing instruments
- No formula sheets or notes

ℹ Grading Reference

Grade	Points Required	Percentage
1 (Excellent)	45-50	90-100%
2 (Very Good)	39-45	77-90%
3 (Good)	32-39	63-77%
4- (Pass)	23-32	45-63%
5-6 (Fail)	0-23	0-45%

Note: Passing grade requires at least 23 points (45%).

Problem 1: E-Commerce Platform Optimization [28 pts. total]

An online marketplace analyzes its pricing and demand relationships for a new product category. Market research reveals strategic information about customer behavior and cost structures.

Part A: Demand and Revenue Analysis

The demand function is linear with a maximum willingness to pay of 150 currency units (CU) when no units are sold. At a price of 30 CU, customers would purchase 40 units (Un).

a) Determine the linear demand function $p(x)$ expressing price as a function of quantity. Show all steps. [4 pts.]

b) Show that the revenue function is given by $R(x) = 150x - 3x^2$. Start from your demand function. [3 pts.]

Part B: Cost Structure

The company has fixed costs of 800 CU per month and variable costs that follow the function $V(x) = 2x + 0.5x^2$ where x represents the quantity produced.

c) Express the total cost function $C(x)$ and compute the cost of producing 25 units. [4 pts.]

For verification purposes only: - $p(x) = 150 - 3x$ - $C(x) = 800 + 2x + 0.5x^2$

Part C: Profit Optimization

d) Determine the profit function $P(x)$ and find the quantity that maximizes profit. Use the vertex formula and verify that this is indeed a maximum. [7 pts.]

e) Calculate the break-even points by solving $P(x) = 0$. Explain their significance for the business using complete sentences. [5 pts.]

Part D: Practical Constraints

f) Due to warehouse limitations, the company can only stock a maximum of 20 units at any time. Determine:
   - The profit at this constraint level
   - The price that should be charged at this quantity
   - Whether the constraint is binding (affecting the optimal solution)

   Provide business reasoning for your conclusions. [5 pts.]

Problem 2: Function Analysis and Business Application [22 pts. total]

Consider the function $f(x) = -0.25x^2 + 4x + 5$ which models the daily profit (in hundreds of CU) of a restaurant based on the number of staff members x.

Part A: Function Properties [12 pts.]

a) Determine the domain that makes sense in this business context. Explain your reasoning using complete sentences. [2 pts.]

b) Find the vertex of the function using the vertex formula $x = -\frac{b}{2a}$. Show your calculation and interpret its meaning for the restaurant. [4 pts.]

c) Determine where the profit equals zero (x-intercepts). Use the quadratic formula and explain what these points represent for the business. [3 pts.]

d) The restaurant currently employs 12 staff members. Calculate the current profit and determine how many additional staff would optimize profit. [3 pts.]

Part B: Transformations and Composition [10 pts.]

The restaurant plans to expand to a tourist location where: - All costs increase by 20% (affecting the entire profit function) - An additional fixed cost of 300 CU per day is incurred

e) Write the transformed profit function $g(x)$ for the tourist location incorporating the cost increase and additional fixed costs. Show the transformation steps. [3 pts.]

f) If the minimum acceptable daily profit is 500 CU (5 hundreds), determine the range of staff numbers that achieve this for:

- The original location
- The tourist location

Show your work algebraically. [4 pts.]

g) The company uses a staffing agency that provides workers according to the function $w(d) = 2d + 6$, where d is the number of days in advance the request is made.

Express the profit as a composite function $(f \circ w)(d)$ for the original location and evaluate the profit when ordering 3 days in advance. [3 pts.]