Mini-Mock Exam 02: Foundations & Equations

BFP Mathematics Course
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Name: _____

Reading Time: 10 minutes

Working Time: 90 minutes

Permitted Aids:

- Calculator (non-programmable without graphing capabilities)
- Drawing instruments
- No formula sheets or notes

ℹ Grading Reference

Grade	Points Required	Percentage
1 (Excellent)	45-50	90-100%
2 (Very Good)	39-45	77-90%
3 (Good)	32-39	63-77%
4- (Pass)	23-32	45-63%
5-6 (Fail)	0-23	0-45%

Note: Passing grade requires at least 23 points (45%).

Problem 1: Algebraic Foundations & Applications [18 pts. total]

A small business analyzes its monthly costs and revenue. The owner discovers that certain algebraic relationships govern the business operations.

Part A: Expression Manipulation

a) Simplify the following expression completely:

$$\frac{x^3 - 8}{x^2 - 4} \cdot \frac{x + 2}{x^2 + 2x + 4}$$

Show all steps of your work. [4 pts.]

b) The monthly overhead costs follow the pattern $C = 2^{n+1} + 2^n$ where n represents the number of months since opening. If the total overhead after n months equals 192 currency units (CU), determine the value of n. [3 pts.]

c) Rationalize and simplify:

$$\frac{2}{\sqrt{7} - \sqrt{3}} + \frac{1}{\sqrt{7} + \sqrt{3}}$$

[4 pts.]

Part B: Applied Problem Solving

d) The company's profit growth rate is modeled by the expression $(x + 2)^3$. Use pascals triangle to expand this expression completely. [3 pts.]

e) If $\log_2(sales) = 3\log_2(5) + \log_2(8) - 2\log_2(10)$, determine the exact value of sales. [4 pts.]

Problem 2: Systems of Equations & Business Applications [20 pts. total]

A manufacturing company produces two types of products: Standard (S) and Premium (P). The production process involves various constraints and relationships.

Part A: Linear Systems

The following production constraints apply:

- Labor hours: 3S + 5P = 205 hours per week
- Material costs: 20S + 35P = 1400 CU per week
- The company must produce at least 10 units of each product

a) Determine how many units of each product the company produces per week using the elimination method. Show all steps and comment whether the solution is feasible. [5 pts.]

b) If the profit per unit is 15 CU for Standard and 25 CU for Premium products, calculate the total weekly profit. [2 pts.]

Part B: Quadratic Applications

c) The demand for Standard products follows the equation $D = -2p^2 + 40p - 150$, where p is the price in CU and D is the demand in units.

i) Find the discriminant and explain what it tells us about the pricing options. [3 pts.]

ii) Determine all prices at which demand equals zero. [3 pts.]

Part C: Complex Equations

d) The production efficiency E (as a percentage) after t hours of operation follows:

$$\frac{100}{t} + \frac{100}{t+3} = 35$$

Determine the time t when this efficiency level is achieved. State any domain restrictions first. Verify which solution(s) are valid in the business context by commenting the solution. [4 pts.]

e) The growth of the company's market share M (in percent) follows $\sqrt{M+16} = M - 4$. Solve for M and verify which solution(s) are valid in the business context by commenting the solution. [3 pts.]

Problem 3: Exponential Growth & Complex Word Problems [12 pts. total]

A startup company is analyzing its growth patterns and investment strategies.

Part A: Exponential Models

a) The company's user base grows according to $U = 1000 \cdot 2^{t/3}$, where t is time in months.

   i) How many users will the company have after 9 months? [2 pts.]

   ii) When will the user base reach 16,000 users? Show your work using logarithms. [3 pts.]

Part B: Investment Analysis

b) The company has two investment options:
   - Option A: Grows at 6% annually
   - Option B: Grows according to the formula $V = P \cdot e^{0.05t}$

   If both start with an initial investment of 10,000 CU, determine which option yields more after 5 years by showing each calculation explicit. [4 pts.]

Part C: Combined Application

c) The company's revenue R (in thousands of CU) and costs C (in thousands of CU) after x months follow:
   - Revenue: $R = 12 \cdot 2^x + 4$
   - Costs: $C = 4 \cdot 2^x + 28$

   Determine the break-even month x. You may leave your answer in logarithmic form if exact. [3 pts.]