Mini-Mock Exam 01: - Business Foundations

BFP Mathematics Course
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Overview

This assessment is a tool for us to understand your current mathematical strengths and to identify areas where we can focus our efforts. Please read these instructions carefully:

- Total Time: Approximately 140 minutes
- No Grades: This test will not be graded. Its purpose is purely diagnostic. Be honest about what you know and don't know.
- Solve or Explain: Your primary goal is to solve each task. If you get stuck or cannot solve a task, use the space provided to write a short, clear sentence explaining why. Examples: "I have forgotten the quotient rule for derivatives," "I don't know how to set up a transition matrix," or "I am unsure what 'asymptote' means." This information is extremely valuable.
- Confidence Scale: For each task, please circle a number from 1 (not confident at all) to 7 (very confident) that reflects how you feel about your ability to tackle that specific task.
- Allowed Materials: You may use a scientific calculator.

Section 1: Mathematical Foundations & Algebra

Suggested time: approximately 15 minutes.

Task 1.A: Manipulating Terms

Confidence (circle one): 1 2 3 4 5 6 7

Simplify the expression completely and show your steps:

$$\left(\frac{x^4\,y}{x^{-2}\,y^3}\right)^2 \cdot y$$

Task 1.B: Business Application with Percentages

Confidence (circle one): 1 2 3 4 5 6 7

A company buys a product for a net price of €150. The company adds a 40% markup to determine the selling price. The final selling price must also include a 19% Value Added Tax (VAT). What is the final gross selling price?

Task 1.C: Logarithms & Scientific Notation

Confidence (circle one): 1 2 3 4 5 6 7

a) Convert 3.2×10^{-4} to decimal form.
b) Solve for x: $\log_2(x) + \log_2(4) = 5$.

Task 1.D: Factorization & Absolute Value

Confidence (circle one): 1 2 3 4 5 6 7

a) Factor completely: $x^2 - 5x + 6$.
b) Solve: $|2x - 3| = 7$.

Task 1.E: Number Systems

Confidence (circle one): 1 2 3 4 5 6 7

Classify each as $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, or $\mathbb{R}$ (choose all that apply):

a) $\sqrt{2}$
b) $-\frac{5}{2}$
c) $0.\overline{3}$

Section 2: Equations & Basic Functions

Suggested time: approximately 25 minutes.

Task 2.A: System of Linear Equations

Confidence (circle one): 1 2 3 4 5 6 7

A small company produces two types of desks: Standard and Premium. A Standard desk requires 2 hours of assembly and 1 hour of finishing. A Premium desk requires 3 hours of assembly and 2 hours of finishing. The company has 180 assembly hours and 100 finishing hours available each week. Let x be the number of Standard desks and y be the number of Premium desks. Set up a system of two linear equations and solve for x and y so that all available hours are used exactly.

Task 2.B: Quadratic Functions in Business

Confidence (circle one): 1 2 3 4 5 6 7

The profit P (in thousands of €) for selling x hundred units is

$$P(x) = -2x^2 + 20x - 32.$$

a) Find the break-even points (where profit is zero).
b) Determine the number of units that maximizes the profit.

Task 2.C: Linear Inequality

Confidence (circle one): 1 2 3 4 5 6 7

Solve the inequality $3x - 5 \leq 2x + 7$ and represent the solution set in interval notation.

Task 2.D: Exponential Equation

Confidence (circle one): 1 2 3 4 5 6 7

Solve for x:

$$5 \cdot 2^{x-1} = 40.$$

Section 3: Advanced Functions & Analysis

Suggested time: approximately 20 minutes.

Task 3.A: Analysis of a Rational Function

Confidence (circle one): 1 2 3 4 5 6 7

Given

$$f(x) = \frac{2x - 4}{x + 1}.$$

a) State the domain of f.
b) Find any x- and y-intercepts.
c) Find the equations of the vertical and horizontal asymptotes.

Task 3.B: Trigonometric Functions in Sales

Confidence (circle one): 1 2 3 4 5 6 7

The quarterly sales of ice cream (in thousands of units) can be modeled by

$$S(t) = 15 \sin\left(\frac{\pi}{2}t - \frac{\pi}{2}\right) + 25$$

where t is time in quarters ($t = 1$ is the end of the first quarter).

a) What are the sales at the end of the second quarter ($t = 2$)?
b) What is the maximum sales volume predicted by this model?

Task 3.C: Exponential/Logarithmic Model

Confidence (circle one): 1 2 3 4 5 6 7

For $f(x) = A \cdot e^{kx}$, suppose $f(0) = 3$ and $f(2) = 12$.

a) Find A and k.
b) State whether the function is growing or decaying and give the continuous growth rate.

Task 3.D: Composition, Domains, and Inverse

Confidence (circle one): 1 2 3 4 5 6 7

Let $g(x) = \sqrt{x - 1}$ and $h(x) = \ln(x)$.

a) Find the domain of $g \circ h$ and of $h \circ g$.
b) Find the inverse of $f(x) = \frac{2x-1}{x+3}$ and state its domain.

Section 4: Differential Calculus

Suggested time: approximately 25 minutes.

Task 4.A: Derivative Rules & Tangent Line

Confidence (circle one): 1 2 3 4 5 6 7

a) Differentiate $y = (x^2 + 1) \cdot e^{3x}$.
b) Compute $y'(0)$ for $y(x) = \frac{\sin(x)}{x^2+1}$.
c) Find the equation of the tangent line to $y = \sqrt{2x + 1}$ at $x = 4$.

Task 4.B: Limits, Continuity, and Difference Quotient

Confidence (circle one): 1 2 3 4 5 6 7

a) Compute $\lim_{x \to 2} \frac{x^2-4}{x-2}$.
b) Let $f(x) = \frac{x^2-1}{x-1}$ for $x \neq 1$ and $f(1) = 3$. Is f continuous at $x = 1$? Justify.
c) For $f(x) = x^2 - 3x$, compute the difference quotient $\frac{f(x+h)-f(x)}{h}$ and simplify.

Task 4.C: Extrema and Inflection

Confidence (circle one): 1 2 3 4 5 6 7

Consider $f(x) = x^3 - 6x^2 + 9x$.

a) Find all critical points and classify them (local max/min) using derivatives.
b) Find any inflection points and intervals of concavity.

Task 4.D: Recovering a Function from its Derivative (Micro)

Confidence (circle one): 1 2 3 4 5 6 7

Given $f'(x) = 3x^2 - 4x + 1$ and $f(0) = 2$, find $f(x)$.

Section 5: Integral Calculus & Applications

Suggested time: approximately 20 minutes.

Task 5.A: Total Profit from Marginal Profit

Confidence (circle one): 1 2 3 4 5 6 7

The marginal profit is

$$P'(x) = -3x^2 + 24x + 27,$$

where x is the number of units sold. Compute the total change in profit from the 4th to the 10th unit using a definite integral.

Task 5.B: Financial Mathematics

Confidence (circle one): 1 2 3 4 5 6 7

An investment of €5,000 earns 4% interest per year, compounded annually. What is the future value after 10 years?

Task 5.C: Integration Techniques & Area

Confidence (circle one): 1 2 3 4 5 6 7

a) Compute the indefinite integral $\int \frac{2x}{x^2+1}\,dx$.
b) Evaluate $\int_0^1 x \cdot e^{x^2}\,dx$ (use substitution).

Section 6: Probability Theory

Suggested time: approximately 20 minutes.

Task 6.A: Conditional Probability

Confidence (circle one): 1 2 3 4 5 6 7

A factory has two machines, A and B. Machine A produces 60% of output with a 3% defect rate; Machine B produces 40% with a 5% defect rate.

a) Draw a tree diagram to represent this situation.
b) What is the probability that a randomly selected item is defective?
c) Given an item is defective, what is the probability it came from Machine A?

Task 6.B: Combinatorics

Confidence (circle one): 1 2 3 4 5 6 7

A department has 10 employees. A project team with three distinct roles is needed: project leader, main developer, and tester. How many different teams can be formed if no one can hold more than one role?

Task 6.C: Binomial Distribution

Confidence (circle one): 1 2 3 4 5 6 7

Let $X \sim \text{Bin } (n = 20, p = 0.3)$.

a) Compute $P(X = 5)$.
b) Find $E[X]$ and $\text{Var}(X)$.
c) Interpret the expectation in a business context.

Task 6.D: Contingency Table & Predictive Values

Confidence (circle one): 1 2 3 4 5 6 7

A screening test has sensitivity 90% and specificity 95%. In a population with 5% prevalence:

a) Build a 2×2 contingency table per 10,000 people.
b) Compute Positive Predictive Value and Negative Predictive Value.

Section 7: Statistics & Linear Algebra

Suggested time: approximately 15 minutes.

Task 7.A: Descriptive Statistics

Confidence (circle one): 1 2 3 4 5 6 7

Monthly revenue (in thousands of €) over six months: 12, 15, 14, 18, 20, 23. Compute the mean and the sample standard deviation.

Task 7.B: Transition Matrices

Confidence (circle one): 1 2 3 4 5 6 7

Two streaming services, StreamA and StreamB, compete for a fixed market. Currently: StreamA has 70%, StreamB has 30%. Each month:

- StreamA retains 80% of its customers, 20% switch to StreamB.
- StreamB retains 90% of its customers, 10% switch to StreamA.

a) Write the transition matrix T for this process.
b) Calculate the market share after one month.
c) Calculate the market share after three months

Task 7.C: Descriptive Stats—Median & IQR

Confidence (circle one): 1 2 3 4 5 6 7

For the dataset: 8, 12, 12, 13, 15, 18, 21

a) Compute the median and the interquartile range (IQR).
b) Which summary (mean vs median) is more robust to outliers? Briefly explain.

Task 7.D: Matrix Inverse & Solving a System

Confidence (circle one): 1 2 3 4 5 6 7

a) Find the inverse of $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ if it exists.
b) Solve $A\mathbf{x} = \mathbf{b}$ for $\mathbf{x}$, where $\mathbf{b} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$.