Tasks 06 - Systems of Linear Equations

Mathematics for Master Students

These problems accompany Session 06 of the tutorial, the last session set. On the website, every problem has a fully worked solution you can expand to check yourself. The PDF version contains only the problems, so you can print it and use it as a worksheet.

The problems are staged: the first two are warm-ups, the middle ones drill the core techniques (substitution, elimination, Gaussian elimination on the augmented matrix, and reading off how many solutions a system has), and the last two practice the mock-exam skill of translating a business story into a system first. See the Mock Exam. Since this rounds off the tutorial, treat the closing problems as a mock-exam warm-up, and keep the Cheatsheet beside you.

Problem 1: Linear or Not, and Is It a Solution?

a) Decide for each equation whether it is **linear** in its unknowns:

   i) $3x - 2y = 7$
   ii) $x^2 + y = 5$
   iii) $4a + 7b - c = 0$
   iv) $xy = 12$
   v) $\sqrt{x} + y = 1$

b) Consider the system $2x + y = 8$ and $x - y = 1$. Check whether each pair is a solution:

   i) $(x, y) = (3, 2)$
   ii) $(x, y) = (4, 0)$

Problem 2: A Quick Substitution

A depot ships goods in small crates (x) and large crates (y). Two conditions fix the plan:

$$y = 2x - 1, \qquad 3x + y = 14.$$

One equation is already solved for y, so use substitution.

Problem 3: One System, Two Methods

Solve the system

$$3x + 2y = 14, \qquad x - y = 3$$

twice (once by substitution and once by elimination) and confirm the two roads reach the same point.

Problem 4: Gaussian Elimination in Three Unknowns

A workshop makes three product types in quantities x, y, z and wants to use every resource fully:

- **Packing:** one box per unit, 6 boxes available $\Rightarrow x + y + z = 6$
- **Labour:** $1, 2, 3$ hours per product, 11 hours available $\Rightarrow x + 2y + 3z = 11$
- **Machine:** $2, 1, 1$ hours per product, 9 hours available $\Rightarrow 2x + y + z = 9$

Write the augmented matrix, reduce it to triangular form by forward elimination, solve by back-substitution, and verify.

Problem 5: How Many Solutions?

For each 2×2 system, decide whether it has a **unique** solution, **no** solution, or **infinitely many**. Where there are infinitely many, describe *all* solutions with a parameter t.

a) $x + y = 5, \qquad x - y = 1$
b) $x + 2y = 4, \qquad 2x + 4y = 10$
c) $x - 3y = 2, \qquad 2x - 6y = 4$

Problem 6: Market Equilibrium

For a product at price p (in €), monthly demand and supply (in units) are

$$\text{demand: } q = 90 - 3p, \qquad \text{supply: } q = 2p - 10.$$

Find the equilibrium price and quantity (the point where the market clears) by solving the system.

Problem 7: A Blending Plan

A factory produces two blends, x and y (in batches), from two raw materials. Each batch of blend x uses 3 units of material A and 5 units of material B; each batch of blend y uses 4 units of A and 2 units of B. This week 44 units of A and 50 units of B are available and must be used in full:

$$3x + 4y = 44, \qquad 5x + 2y = 50.$$

Find the number of batches of each blend.