Mock Exam

Mathematics for Master Students

This is the **voluntary mock exam** for the tutorial. It is adapted from the original KLU course exam and covers all six sessions: notation and logic, sets, functions, differentiation, optimization, and linear systems. Nothing here is graded and no result is tracked; the exam only exists so you can rehearse under exam-style conditions before your M.Sc. courses lean on this material.

Suggested conditions. Work through the ten problems in about **90 minutes without notes** first, as if it were the real thing. Only then open the collapsible solutions to mark yourself, and pull out the cheatsheet that matches the problem you are stuck on. The pointer list at the end of this page maps every problem to its session. On the website every problem carries a fully worked solution you can expand; the **PDF version prints only the problems**, so you can use it as a clean exam sheet.

The problems are numbered as in the source exam, not in session order. Each is followed by a worked solution showing the named method, the steps, and a check where the topic bundles one.

Problem 1: Functions (Domain, Composition, Inverse)

A logistics cost model chains two functions,

$$f(x) = 3x - 6 \qquad \text{and} \qquad g(x) = \sqrt{x}.$$

a) State the domain of g, then the domain of the composition $g(f(x))$.
b) Find the inverse $f^{-1}(x)$.
c) Is f convex, concave, or linear?

Problem 2: Sum Equals Product

Find natural numbers x_1, x_2 (with $1 \le i \le 2$) that solve

$$\sum_{i=1}^{2} x_i = \prod_{i=1}^{2} x_i.$$

Problem 3: When Is Total Profit Positive?

A firm makes products $p \in P$ from resources $r \in R$. The notation $\sum_{p \in P}$ sums over every product p in the set P, the set-membership cousin of Session 1's indexed sum $\sum_{i=k}^{m}$ (and $\sum_{r \in R}$ likewise runs over all resources r). With quantity q_p, selling price s_p, resource requirement $a_{r,p}$ (units of resource r per unit of product p), and resource cost c_r, total profit is

$$\pi = \sum_{p \in P} q_p \left(s_p - \sum_{r \in R} a_{r,p}\, c_r \right).$$

Assume every quantity $q_p > 0$ and every resource cost $c_r > 0$. Decide which of the following conditions **guarantee** $\pi > 0$, and justify each:

i) $a_{r,p} = 0$ for all r, p, and $s_p > 0$ for all p.
ii) $s_p - \sum_{r \in R} a_{r,p}\, c_r < 0$ for all p.
iii) $a_{r,p} > 0$ and $s_p > 0$ for all r, p.

Problem 4: Fuel Consumption Conversion

A delivery van imported from the US is rated at **20 miles per gallon**. Your European fleet software needs the figure in **litres per 100 km**. Using $1 \ \text{mile} = 1.6 \ \text{km}$ and $1 \ \text{gallon} = 3.8 \ \text{litres}$, set up the conversion. A formula is sufficient, but give the value as well.

Problem 5: Logical Implications

Let $p(x)$ be the proposition $x^2 = 16$, let $q(x)$ be $x = 4$, and let $r(x)$ be $x = \sqrt{x^2}$, all for real x. Decide which of the following implications hold for **every** real x:

i) $p(x) \Rightarrow q(x)$
ii) $q(x) \Rightarrow r(x)$
iii) $r(x) \Rightarrow p(x)$
iv) $p(x) \ \text{ and } \ r(x) \ \text{ together} \Rightarrow q(x)$
v) $q(x) \Rightarrow p(x)$

Problem 6: Union and Cartesian Product

For the transport modes $A = \{\text{road}, \text{rail}, \text{air}\}$ and $B = \{1, 2, 3\}$, decide which of the following are correct:

i) $A \cup B = \{(\text{road}, 1), (\text{rail}, 2), (\text{air}, 3)\}$
ii) $A \cup B = \{\text{road}, \text{rail}, \text{air}, 1, 2, 3\}$
iii) $A \times B = \{(\text{road}, 1), (\text{rail}, 2), (\text{air}, 3)\}$
iv) $A \times B = \{(\text{road}, 1), (\text{road}, 2), (\text{road}, 3), (\text{rail}, 1), (\text{rail}, 2), (\text{rail}, 3), (\text{air}, 1), (\text{air}, 2), (\text{air}, 3)\}$
v) $\{i \in \mathbb{N} \mid \text{letter number } \ i \ \text{ is 'r' in some } \ a \in A\} \cap \{b \in B \mid b \ \text{ is even}\} = \emptyset$

Problem 7: Set Identities

Let A, B, C be arbitrary sets and let A^c denote a complement. Decide which identities hold for **all** choices of A, B, C:

i) $C \cap (A \cap B) = A \cap (B \cap C)$
ii) $C \cap (A \cup B) = A \cup (B \cap C)$
iii) $C \cup (A \cap B) = A \cup (B \cap C)$
iv) $(A \cap B)^c = A^c \cup B^c$
v) $(A \cup B)^c = A^c \cap B^c$

Problem 8: Profit Maximization

A manufacturer faces the price-demand relation $p(q) = 100 - 2q$ (price in € when q units are sold) and the cost function $C(q) = 20q + 100$ (in €). Find the quantity that maximizes profit, confirm it is a maximum, and state the price and the maximum profit.

Problem 9: Derivative and Monotonicity

For $f(x) = x^2 - 4x + 3$, find the derivative $f'(x)$, then decide which of the following are correct:

i) f is decreasing on $(-\infty, 2]$
ii) f is increasing on $(-\infty, 2]$
iii) f is decreasing on $[2, \infty)$
iv) f is increasing on $[2, \infty)$
v) f is increasing on the set of non-negative real numbers
vi) f is decreasing on the set of non-negative real numbers
vii) f is neither increasing nor decreasing on the whole set of real numbers

Problem 10: Solve the Linear System

Solve the system

$$x - 2y = 2, \qquad \frac{x}{3} - y = \frac{1}{3}.$$

After the Mock Exam

Whatever felt shaky points you straight back to the session that covers it. Each cheatsheet is a one-page refresher, and each self-test quiz gives you a few more reps:

- **Problems 2, 3, 5** (notation, logic, formulas): Cheatsheet 01 and Self-Test 01
- **Problem 4** (unit conversion): pure ratio arithmetic, no cheatsheet needed; the worked solution walks through every step
- **Problems 6–7** (sets): Cheatsheet 02 and Self-Test 02
- **Problem 1** (functions): Cheatsheet 03 and Self-Test 03
- **Problem 9** (differentiation): Cheatsheet 04 and Self-Test 04
- **Problem 8** (optimization): Cheatsheet 05 and Self-Test 05
- **Problem 10** (linear systems): Cheatsheet 06 and Self-Test 06

Rework anything you missed, then you are ready for the math your M.Sc. courses will lean on.